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APPROXIMATE PROBABILISTIC METHODS FOR SURVIVABILITY/VULNERABILITY ANALYSIS OF STRATEGIC STRUCTURES

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The major concepts and methods of probability and statistics that are of special significance to engineering are summarized, with emphasis on the potential applications in strategic structures problems. Approximations to well-established exact mathematical methods of probabilistic analysis were developed; these approximations are necessary for a sensible and effective implementation of the essential concepts in engineering, in the sense that they lead to simplifications that are consistent with the state and quality

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20. ABSTRACT (Continued)

of available information, which (in practice) invariably must include subjective judgments.

Applications in the development of probability-based relationships for the design of strategic structures were stressed; the required relationships can be developed so that the conventional (i.e. deterministic) methods of design may be retained. In particular, these may be in the form of safety factors that can be used in a conventional design format, in order to achieve a desired survivability in terms of a probability of survival. Similar relationships for target planning are also developed, and their significance identified and discussed.

Several examples of strategic structural problems are developed to demonstrate the basic approach.

PREFACE

This report represents the results of a study to develop the current state-of-the-art of probabilistic methods for structural vulnerability/survivability analysis and design. The study was performed for the Strategic Structures Division of the Defense Nuclear Agency, as part of Contract DNA 001-77-C-0177 with N.M. Newmark Consulting Engineering Services.

The study was monitored by Dr. E. Sevin and Captain M. Moore of DNA; their interests and support for the study are greatly appreciated.

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I. INTRODUCTION

The analysis of strategic systems, either for the purpose of evaluating the survivability/vulnerability of a hardened system against a postulated enemy attack, or for evaluating the effectiveness (i.e. kill potential) of a targeting plan against an enemy installation, involves many sources of uncertainty. In this light, the survivability or weapon effectiveness cannot be determined or assured in absolute terms; realistically, measures of assurance may be given only in terms of probability. For these reasons, the methods and concepts of probability and statistics are pertinent and can be useful in many aspects of strategic planning and design. How probabilistic methods may be used most effectively in different types of strategic problems or for different objectives, however, is not always clear. From the standpoint of design and planning of strategic structures and facilities, the problems are similar to those of conventional engineering; on this premise, this report will emphasize the existing concepts and approximate methods of probabilistic analysis that have been developed and implemented in other areas of engineering. The applicability of these methods to strategic structural problems will also be delineated; in particular, approximate probabilistic methods that are particularly appropriate and suitable for formulating probabilistic bases for strategic design will be emphasized.

Probability and statistics are important in engineering analysis and design, to be sure; however, their proper roles should be viewed from the perspective of broad engineering implications and not merely from a mathematical standpoint. For example, the fact that survivability and weapon effectiveness may be assured only in probability terms does not necessarily mean that all analysis and design must be performed probabilistically. In particular, the routine design (i.e. of determining the sizes of structural components) can still be accomplished through the use of conventional design factors, e.g. safety factor or load factors, that are derived on the basis of achieving specified probability of survival.

Mathematically exact or rigorous probabilistic methods are, of course, available; however, the implementation of these exact methods to practical problems is invariably difficult or extremely limited. Moreover, in view of the fact that the available state of information in most practical engineering problems are limited and must be supplemented with subjective engineering judgment, the calculational complexity that invariably underlies rigorous methods is seldom justifiable. Approximate methods of analysis that are consistent with the available state of data and information may be more appropriate and effective. The material presented herein, therefore, is limited to an approximate calculational approach that is adequate for engineering purposes. It is believed that the same approximate method is equally adequate and useful for many strategic problems, especially those of strategic structural problems.

1.1 Role of Probability in Engineering

Engineering planning and design in general, and engineering for strategic purposes in particular, are of necessity based on predictions of the real world. Such predictions are invariably based on limited information and observational data, as well as on idealized assumptions; the latter are necessary for reasons of simplicity and practical expediency. In this light, engineering predictions are very seldom perfect; uncertainty, therefore, is unavoidable. As a consequence, the performance (or mission success) of an engineering system or strategic system cannot be assured with absolute certainty. An assurance would be realistically possible only in terms of probability. In this context, probability represents a realistic measure of assurance of performance under conditions of uncertainty. For this purpose, the effects of all sources of uncertainty should be included; to be sure, this must include the uncertainty associated with the imperfection of predictions, as well as those associated with inherent randomness or scatter of observational data.

As with the results of other (deterministic) methods of engineering analysis, which provide synthesized information for purposes of developing "optimal" designs, a probabilistic engineering analysis should also lead to the synthesis of uncertainty information, to supplement purely deterministic analysis. This supplemental information is particularly useful, and even necessary, when the design variables and predictive models underlying the design process contain uncertainty. In other words, probability provides the formal and logical framework for the analysis and treatment of uncertainty, and for the evaluation of the effects of uncertainty on engineering performance and design. These uncertainties include that associated with randomness of available information as well as those associated with errors of prediction underlying the process of engineering design. In this regard, the proper role of probability concepts and methods is supplemental (but important) to an existing deterministic approach for engineering analysis and design. In other words, engineering will basically remain deterministic. Probability and statistics can be most effective if used to complement the existing deterministic technology; namely, by providing the necessary basis and tools for the explicit consideration of uncertainty.

1.2 Objectives and Coverage of Report

This report will summarize and illustrate the current state-of-the-art of probabilistic concepts and methods for engineering pruposes, with special emphasis on the potential applications of these concepts to problems of survivability and vulnerability of strategic structures. Effectiveness in the implementation of these concepts and methods are discussed and illustrated; where alternative concepts and implementation procedures are available or have been proposed, the implications of such alternative procedures will be identified.

As with deterministic methods for engineering analysis and design, simplicity in a probabilistic approach is important for purposes of effective engineering implementation, as well as for ease of understanding. This requirement is emphasized throughout the report. Refinements and mathematically more exact methods are available; however, the implementation of more rigorous methods will be at the expense of complications and difficulty of understanding. For these reasons, mathematical refinements do not always mean practical effectiveness.

The coverage of this report, therefore, is limited to the essential elements of probability and statistics that can effect solutions to practical problems. For this objective, simplification and approximations are necessary, without which the practical usefulness of probability and statistics may remain unrecognized.

The material presented in this report, therefore, were selected and developed on the following premises:

1. Exact methods of probabilistic analysis are not necessarily the most appropriate or effective for engineering purposes, including applications to strategic problems, for the following reasons:

a. Exact analyses are invariably complicated except for the simplest cases.

b. More importantly, the state of available information in most practical applications seldom warrants exact calculations. In otherwords, when the available information is largely based on subjective judgments, mathematical rigor becomes less important; an approximate solution that is consistent with the reliability of the underlying information is more sensible.

2. The introduction of probability concepts will not necessarily require or result in a different method of design and analysis (there may be some misconception on this point). Probability serves to complement or supplement deterministic predictions underlying engineering analysis and design; it can be most effective if used to complement the existing deterministic prediction methods by providing the basis and tools for the explicit consideration of the uncertainty underlying such predictions.

The weaknesses of a purely deterministic approach have been recognized; however, a strictly mathematical and exact probabilistic approach is also limited and fraught with problems in implementation. It is the purpose here to identify what can be done, by way of approximations that are necessary to implement probability most effectively, and also develop the necessary approximate approach with emphasis on problems of survivability/vulnerability of strategic structures.

Illustrations of the concepts and methods developed herein are limited to basic cases; that is, they are limited to the determination of the survival probability or design of structural components. These would include structural systems if the system capacity and associated weapon effects are the information specified. However, the problem of determining the survival probability of a system on the basis of the survival probabilities of its components is not covered; this latter problem requires probabilistic system analysis and is beyond the scope of the present work.

There is some similarity between the basic problem of target planning to achieve a specified kill probability and the problem of design to achieve a survival probability; for this reason, most references to targeting or target planning are intended only to demonstrate this similarity. Undoubtedly, there are many aspects of the targeting problem that are difficult or have no counterpart in survivability design; such problems, of course, are also outside the present scope.

The basic analytical tools for uncertainty evaluation, and analysis of its effect on the probability of survival (or kill) of a strategic system are described and illustrated in Chapter 2. Alternative approaches for implementing probability in engineering for strategic purposes are reviewed in Chapter 3; the implications of each approach are also examined. Also, in Chapter 3, the survivability and design of underground tunnels and of equipment to nuclear weapons effects are discussed in the context of survival (or kill) probability. Chapter 4 summarizes the main results and emphases of the Report, and describes several suggestions for additional study.

II. PROBABILITY, UNCERTAINTY, AND SURVIVABILITY

2.1 Engineering Interpretation of Probability

For engineering purposes, a calculated probability (e.g. of survival) represents a measure of assurance of performance (or mission success) of an engineering system; for this purpose, it should reflect the consequences of all sources of uncertainty (in one form or another) on the performance and design of the system. Invariably, such a calculated probability is based on the logical synthesis of available information (including judgmental information), obtained through established physical relationships for the pertinent problem. In most practical situations, where the pertinent data are extremely limited, a calculated probability would be difficult to justify if interpreted strictly on the basis of the frequency definition of probability, which is based on a large number of repeated observations; experimental statistical verification may be required only of the information for the individual variables that underly the calculated probability. That is, verification may be expected and possible only for the statistical information of the constituent variables, the synthesis of which is the basis of the calculated probability. A calculated probability, obtained in this manner, nevertheless remains useful and significant for engineering application, in the same sense that the result of a conventional deterministic analysis is useful for engineering purposes.

Purposes of a Calculated Probability -- Depending on whether the purpose is the assessment of survivability (or kill) or the formulation of criteria for design, the significance of uncertainty and of the associated probability may be defined and used differently, as described below.

1. Assessment of Survivability -- In the case of assessment, the objective is to seek an estimate of the true probability of survival or kill. Because the parameters (particularly of the mean or median) of the probability distributions used in the determination of the estimate may be in error (arising from the imperfection of the model and/or insufficient data), the estimate would contain uncertainty. This uncertainty has been expressed in terms of a confidence interval (e.g. 90% confidence limits, TRW 1977). In view of the fact that the uncertainties in the parameters must invariably be assessed subjectively, a confidence statement would be much too precise, which may not be warranted in light of the subjective basis for such statements.

In the case of assessment, the calculated probability may be limited to the consequence of the inherent variability, representing the uncertainty due to randomness only. Model imperfection and lack of sufficient data will introduce uncertainty into the estimate of the true probability; this uncertainty may be expressed in terms of a range of possible values within which the true probability will lie. In light of the fact that the basis for the range is often purely subjective, expressing this range simply in terms of its dispersion (e.g. its standard deviation) would be less precise but is more consistent with the subjective basis for its estimate.

2. Formulation of Criteria for Design -- In the development of criteria for design to insure a given probability of survival, the objective is to formulate a design that will cover all sources of uncertainty, including those due to "ignorance." In otherwords, in design under conditions of uncertainty, the objective is to make proper allowance (e.g. through a factor of safety) to cover all sources of uncertainty including those due to prediction error or ignorance, such that a specified level of survivability is achieved. In this latter case, therefore, the inherent variability as well as uncertainty due to model imperfection and insufficient data may be combined in determining the proper factors for design.

For the purpose of design formulation or decision-making, unambiguity would be desirable or essential. For this latter purpose, therefore, developing the required design criteria based on a point estimate of the survival probability reflecting the combined effects of all sources of uncertainty will avoid any ambiguity.

In other words, for the formulation of design, a point estimate of the survival probability, or kill, would be preferable (as opposed to an interval estimate) for reasons of uniqueness and/or unambiguity; whereas in the assessment of the true probability, a measure of error or dispersion on the estimated probability may also be specified.

2.2 Basic Considerations and Formulations

The survivability or vulnerability of a structure or facility to a given environment is, of course, a matter of the available resistance of the structure relative to the applied weapon effect. If the actual resistance and weapon effect can be precisely predicted, there would be no question about the assurance of survivability; conversely, if there is precise information on the weapon effect and the capacity of an enemy installation, the weapon system necessary to insure destruction may also be ascertained. However, in the presence or under conditions of uncertainty, the survival or destruction of a given structure cannot be assured in absolute terms; realistically, it is only possible to insure survival or destruction in terms, or to the extent, of a given probability.

Because of uncertainty, the resistance R and weapon effect S may be described as random variables. Complete description of these random variables may be accomplished with the respective probability density functions (PDF) $f_R(r)$ and $f_S(s)$, as shown graphically in Fig. 1.

As random variables, the actual values of R and S, therefore, may be specified only with their respective probabilities; for example,

$$P(r_0 \geq R \geq r_0 + dr) = f_R(r_0)\, dr;$$

or,

$$P(R \leq r_0) = \int_0^{r_0} f_R(r)\, dr.$$

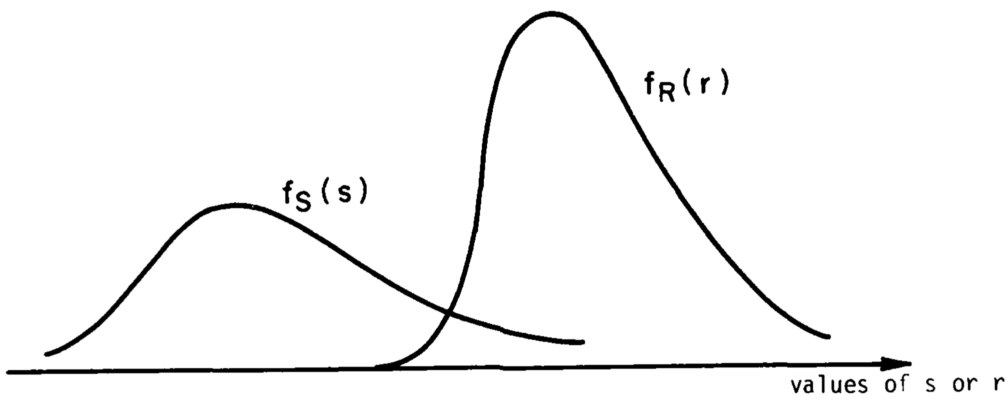


FIG. I DESCRIPTION OF R AND S IN TERMS OF PDF

where, $f_R(r)$ is the probability density function, or PDF, of R.
In this light, survival then is the event $(R > S)$; conversely, failure is the event $(R \leq S)$. Accordingly, the probability of failure is,

$$p_F = \int_0^\infty F_R(s) \cdot f_S(s)\, ds \tag{1}$$

$$= \sum_{\text{all } s_i} F_R(s_i) \cdot \Delta F_S(s_i) \tag{1a}$$

where, $F_R(s)$ is the cumulative probability of $(R \leq s)$. Alternatively, the failure probability may be expressed also as,

$$p_F = \int_0^\infty [1 - F_S(r)] \cdot f_R(r)\, dr \tag{2}$$

$$= \sum_{\text{all } r_i} [1 - F_S(r_i)] \cdot \Delta F_R(r_i) \tag{2a}$$

where, $F_S(r)$ is the cumulative probability of $(S \leq r)$.

The probability of survival then is the complimentary probability or

$$p_S = 1 - p_F \tag{3}$$

As represented in Eq. 1 or 2, the probability of failure p_F, (or of kill) is related to the overlapping region (actually the convolution of probabilities) between $f_R(r)$ and $f_S(s)$ as indicated in Fig. 2. Accordingly, the probability of failure is a function of the relative positions between $f_R(r)$ and $f_S(s)$ as may be seen in Fig. 2;

in particular, observe that as the relative positions between $f_R(r)$ and $f_S(s)$ increases, the probability of failure decreases -- $p_{F2} < p_{F1}$ in Fig. 2.

The probability of failure (and also of survival) depends also on the degree of dispersion in the possible values of R and S; this may be observed succinctly in Fig. 3. Observe that for the same relative positions between $f_R(r)$ and $f_S(s)$, the overlapping region (and thus p_F) increases as the dispersion in the possible values of R and S increases -- compare the overlapping region of the solid curves with that of the dashed curves in Fig. 3.

In general, the discretized forms represented by Eqs. 1a and 2a would be useful. However, for certain probability distributions of R and S, closed-form analytical results are possible and have been derived. In particular, the lognormal distribution is of special interest as it has been used widely in strategic problems; accordingly, specific results involving the lognormal, as well as the normal, distributions for R and S are illustrated herein.

Suppose that R and S are statistically independent random variables, and their distributions are individually lognormal with the following parameters:

Parameters of R	Parameters of S
$\tilde{r}$ = median R	$\tilde{s}$ = median S
ζ_R^2 = variance of ln R	ζ_S^2 = variance of ln S

where, ln stands for natural logarithm; then, the probability of survival p_S, is

$$p_S = \Phi\left(\frac{\ln \tilde{r}/\tilde{s}}{\sqrt{\zeta_R^2 + \zeta_S^2}}\right) = \Phi\left(\frac{\ln \tilde{\theta}}{\zeta}\right) \tag{4}$$

where:

$\tilde{\theta} = \tilde{r}/\tilde{s}$, is the "median safety factor."

$\zeta = \sqrt{\zeta_R^2 + \zeta_S^2}$

$\Phi(x)$ = the standard normal probability

Alternatively, if R and S are statistically independent normal variates, the corresponding survival probability would be

$$p_S = \Phi\left(\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right) = \Phi\left(\frac{\bar{\theta} - 1}{\sqrt{\bar{\theta}^2 \delta_R^2 + \delta_S^2}}\right) \tag{5}$$

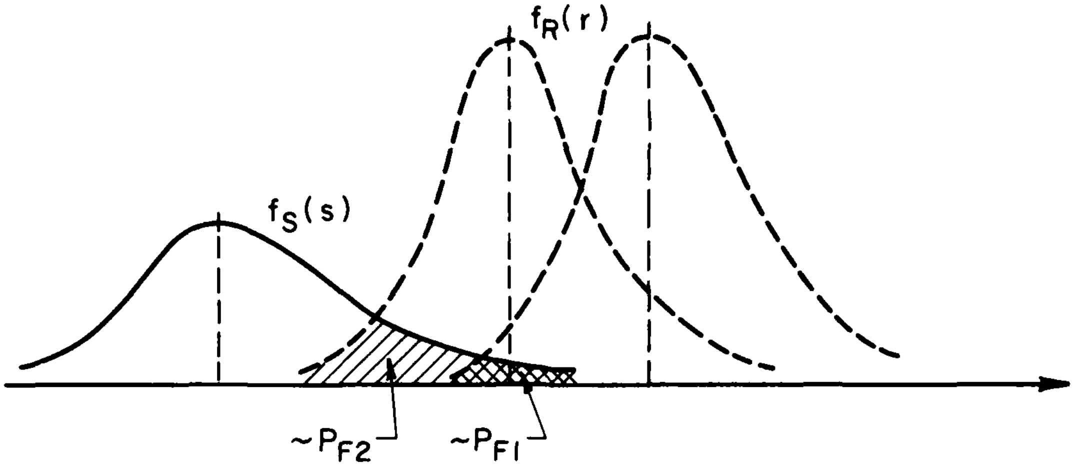


FIG. 2 EFFECT OF RELATIVE POSITIONS OF $f_R(r)$ AND $f_S(s)$ ON P_F

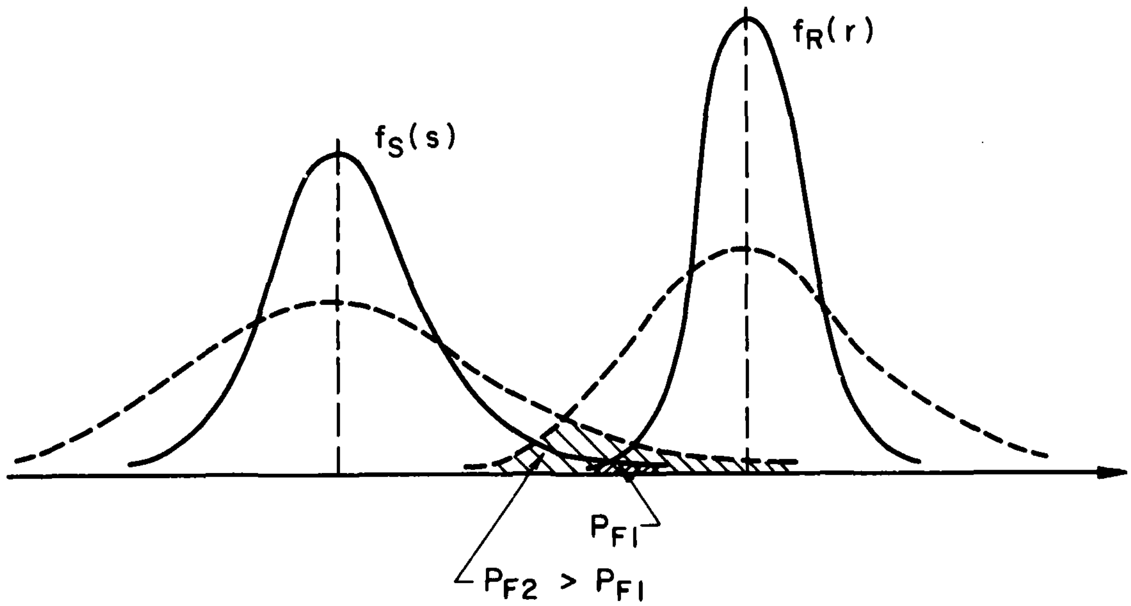


FIG. 3 EFFECT OF DISPERSION ON P_F

where: μ_R = mean resistance

μ_S = mean weapon effect

σ_R, σ_S = standard deviations of R and S, respectively.

Ω_R, Ω_S = coefficient of variation (c.o.v.) of R and S, respectively.

$\bar{\theta} = \mu_R/\mu_S$, the "mean safety factor".

Other forms of distributions for R and/or S, of course, may also be used to calculate the appropriate survival probability. Properly, the most appropriate distributions for R and S should be used; selection of these distributions, however, often have to be based largely on judgment. This may be guided by certain physical or logical considerations; for example, if the process is largely multiplicative, the lognormal distribution may be appropriate, whereas if it is largely additive, the normal distribution may be more appropriate. Finally, if there is sufficient data to indicate or favor particular distributions for R or S, then such distributions ought to be used; in practice, however, it is seldom that there is sufficient data for this purpose. For general distributions of R and S, numerical integration of Eq. 1 or 2 may be necessary to evaluate the probability of survival or failure; in such cases, the discretized form of these equations, namely Eq. 1a or 2a, would be convenient.

Probability-Based Design Relationships -- In the context used herein, "design" refers to the determination of the required capability of a structure or facility to resist a given weapon effect. This may be the determination of the median structural resistance necessary to insure survivability against a given weapon effect; similarly, it could also be the determination of the weapon size necessary to inflict a level of damage on an enemy installation. Since it is not possible to give absolute assurance of survivability (or kill), the objective of a design is to insure survivability (or mission success) in terms of probability. Such designs, however, can be accomplished without probabilistic analysis; i.e., for this purpose, certain relationships or probability-based criteria has to be developed, on the basis of which the routine process of design or sizing of a given system can be carried out with the usual conventional (deterministic) procedure.

The development of the necessary design relationships may be illustrated also for the lognormal or normal distributions for R and S as follows:

Referring to Fig. 2, design may be viewed as the determination of the position of $f_R(r)$ sufficiently far from the position of $f_S(s)$ so that an acceptable probability of failure (or survival) is achieved. The relative positions between $f_R(r)$ and $f_S(s)$ may be measured by the ratio of the medians, namely $\theta = \tilde{r}/\tilde{s}$ which may be called the "median factor of safety." In otherwords, a specified probability of survival would be achieved if the required median resistance is given as

$$r \quad s$$

where s = the median weapon effect, and = the required median safety factor representing the relative positions between $f_R(r)$ and $f_S(s)$. The problem of design, therefore, can be reduced to the determination of the appropriate median safety factor, , in order to achieve an acceptable probability of survival p_S. Development of the relationship between and p_S, therefore, is necessary which can be accomplished as follows.

Inversion of Eq. 4 for , we obtain

$$= e \qquad (6)$$

where = $^{-1}(p_S)$, the value of the standard normal variate at probability p_S.

It may be observed from Eq. 6, that for given values of , the required median safety factor is a function of the survival probability p_S. This relationship is plotted in Fig. 4 for various values of .

Similarly, if the distributions of R and S are respectively normal, the required design relationship may also be developed as follows: In this case, inverting Eq. 5 for the mean safety factor gives

$$= \frac{1 + \sqrt{{}^2_R + {}^2_S - {}^{2\,2\,2}_{R\,S}}}{1 - {}^{2\,2}_R} \qquad \frac{1 + \sqrt{{}^2_R + {}^2_S}}{1 - {}^{2\,2}_R} \qquad (7)$$

where again, = $^{-1}(p_S)$, and

$_R$, $_S$ = the coefficient of variation of R and S, respectively.

Eq. 7 is presented graphically in Fig. 5.

It is significant to observe from Figs. 4 and 5, or Eqs. 6 and 7, that the required safety factor is a function of the c.o.v. representing the total dispersive uncertainty underlying the design. For this reason, the quantitative determination and evaluation of credible c.o.v.'s are all-important in the development of proper bases for design; these should include in particular, the c.o.v.'s associated with prediction errors.

Generalizations -- The resistance R and the weapon effect S may, respectively, be functions of several random variables, in which the probability distributions and associated parameters of the individual variables may be known or assumed; in such cases, the distributions of R and S will obviously depend on those of the constituent variables. Theoretically, the required distributions for R or S (in this case) may be derived from those of the respective constituent variables. For example, if

$$R = g(R_1, R_2, \ldots, R_n) \qquad (8)$$

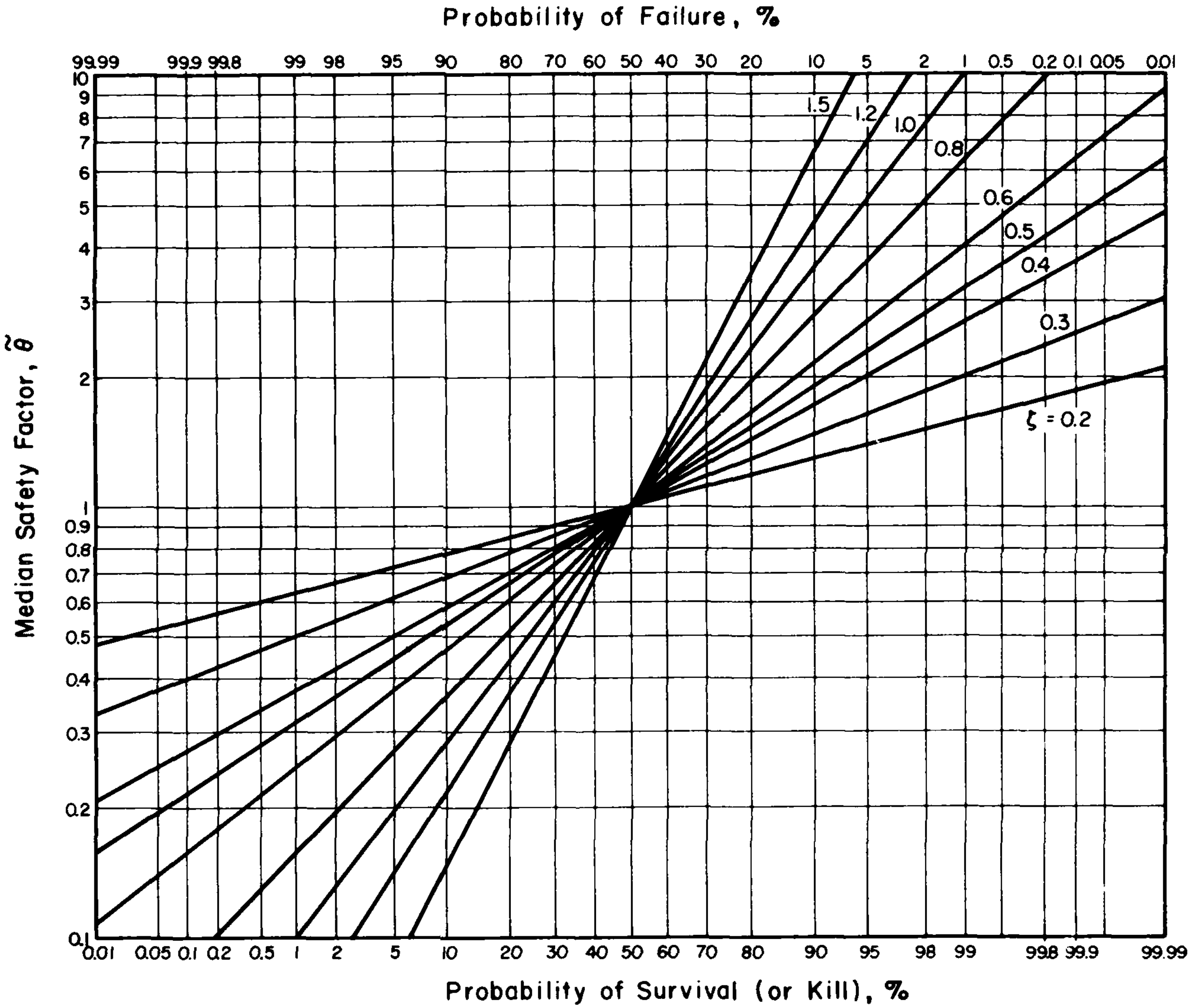


FIG. 4 SAFETY FACTOR VS SURVIVAL PROBABILITY RELATIONSHIP (LOGNORMAL R & S)

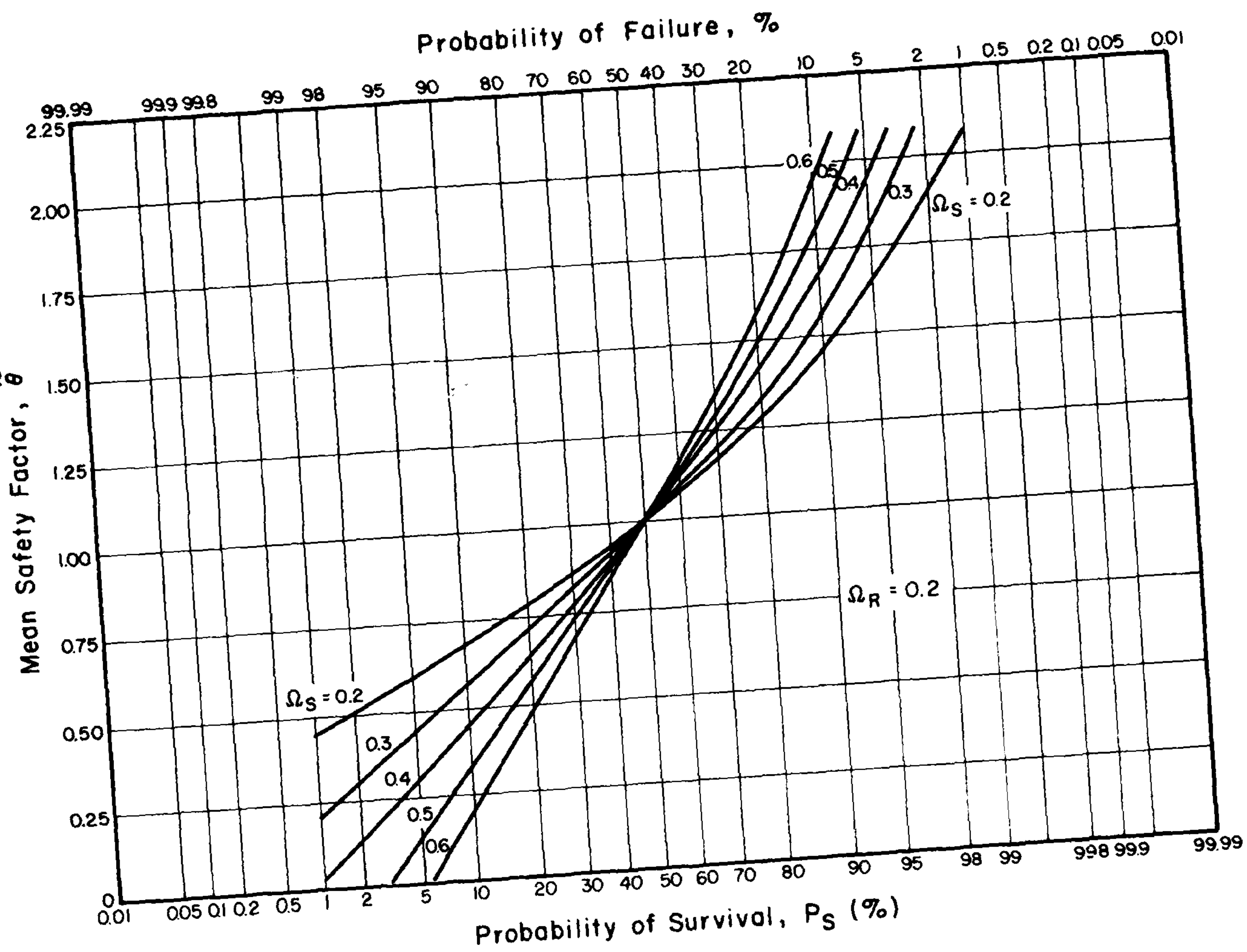


FIG. 5 SAFETY FACTOR VS SURVIVAL PROBABILITY RELATIONSHIP (NORMAL R & S)

where the density functions of the constituent variables $R_1, R_2, \ldots, R_n$ are given, the distribution function of R can be shown (for statistically independent $R_1, R_2, \ldots, R_n$) to be

$$F_R(r) = \int \cdots \int_{\{g(r_1,\ldots,r_n) \leq r\}} f_{R_1}(r_1)\, f_{R_2}(r_2) \ldots f_{R_n}(r_n) dr_1 \ldots dr_n \qquad (9)$$

The distribution function of S may similarly be derived as a function of the individual weapon effect variables. The required probability of failure, or survival, may then be obtained through numerical integration using Eq. 1 or 2.

Deriving the probability distributions of R and S as functions of the distributions of the respective constituent variables (i.e., through Eq. 9) is laborious; theoretically it may be performed numerically. However, the probability distributions of the constituent variables, such as those of $R_1, R_2, \ldots, R_n$, are generally not that well known; for this reason, any effort to derive $F_R(r)$ and/or $F_S(s)$ rigorously in the manner of Eq. 9 may not always be warranted. Of course, if the probability distributions of the constituent variables, such as $f_{R_1}, f_{R_2}, \ldots, f_{R_n}$ are well established, then the correct derived distribution $F_R(r)$ ought to be derived through Eq. 9. However, it is very seldom that there are cases of this nature in most practical problems.

For practical purposes, the forms of the distribution for $f_R(r)$ and $f_S(s)$ may be prescribed, taking into consideration relevant physical and mathematical considerations. For example, if the functional relation for R, i.e., Eq. 8, is principally a product of several resistance variables, the distribution for R may reasonably be prescribed to be lognormal, by virtue of the central-limit theorem; whereas, if R (or S) is largely the sum of several variables, then its distribution may tend to be normal for the same reason. In this regard, as there are numerous subjective factors that are significant in strategic problems whose affects may be assumed to be multiplicative, the assumption of the lognormal distributions for both R and S, therefore, could be reasonable.

Furthermore, in the range of probabilities of interest to strategic problems, i.e. $0.01 \leq p_S \leq 0.99$, the calculated probabilities are not very sensitive to the prescribed distribution form; for this reason, the probability calculated on the basis of reasonably prescribed distributions may often be sufficient for practical purposes. Thus, for stategic purposes, the probability of survival (or failure) calculated on the basis of judgmentally prescribed form of distributions for R and S may be adequate.

In light of the above considerations, the lognormal or normal distributions are attractive by virtue of the resulting mathematical simplicity; therefore, unless there is evidence to indicate otherwise, either of these two distributions may be used to derive useful results.

Observations -- Irrespective of whether closed-form analytical results can be obtained, or numerical integration is necessary, the survival probability (as illustrated explicitly in Eqs. 4 and 5) may be seen to be a function of the relative positions between $f_R(r)$ and $f_S(s)$ and of the degrees of dispersion in $f_R(r)$ and $f_S(s)$. The relative positions between $f_R(r)$ and $f_S(s)$ may be measured in terms of the ratio of the median values r/s or of the mean values $\bar{r}/\bar{s}$, which may be called the "central factor of safety."

It might be emphasized that the central safety factor = $\bar{r}/\bar{s}$ or = r/s, which is the ratio of the mean or median values of R and S, is a deterministic quantity; its determination, therefore, involves purely deterministic analysis. The standard deviation or coefficient of variation, however, is a statistical quantity; its determination, therefore, requires statistical methods as will be described below. It is important to recognize this difference; i.e. that statistical methods are required only for the purpose of assessing and analyzing the degree of dispersion (representing dispersive uncertainty), whereas existing deterministic methods must still be used to determine the central safety factor. This recognition is important for delineating the proper role of probability concepts in engineering evaluation and design.

2.3 Modelling and Analysis of Uncertainty

2.3.1 Introductory Remarks

In engineering, when we speak of uncertainty we are really concerned with the question of "How well can we predict (or estimate) the state of nature?"; that is, it is the uncertainty underlying one's prediction of the real world that is pertinent. Uncertainty, therefore, arises from one's inability to make a perfect or precise prediction of reality. In this sense, uncertainty may be due to inherent randomness or to the imperfection in the method of prediction; i.e. both the randomness in the physical process and any imperfection in the prediction of the process contribute to the total degree of uncertainty.

If the underlying phenomenon is random, prediction is usually limited to the estimation of a central value (e.g. the mean or median) and associated standard deviation or coefficient of variation. Seldom will there be information and data sufficient to determine the complete probability distribution, such as its PDF, and thus when necessary, the PDF may be prescribed judgmentally, taking into consideration relevant physical or mathematical factors as mentioned earlier.

Of first order importance is the uncertainty associated with error in the prediction of the central value (mean or median); although there may be error also in the estimated standard deviation or c.o.v., the uncertainty associated with this latter error is of secondary importance. In other words, aside from the uncertainty inherent with the randomness of the physical process, there is also uncertainty associated with the inaccuracy in the prediction which may be limited to the error in the estimation of the central value.

Potential error in the estimate of the central value may contain a systematic component (bias) and a dispersive component (or random error). The systematic error is due to factors that will tend to bias the predicted estimate consistently in one direction, whereas the random error will contribute an uncertainty representing the range of possible values within which the correct central value may lie.

2.3.2 Types and Sources of Uncertainty

As alluded to above, uncertainty may arise from (i) basic randomness, or (ii) error of prediction. Each type of uncertainty may be described further as follows:

Uncertainty due to Randomness -- Uncertainty is associated with randomness because the exact realization of a physical phenomenon is not completely predictable. The conceivable or possible realizations may be described only in terms of a range of possibilities with their respective relative likelihoods of occurrence (e.g. with a probability density function). In other words, if the state of nature is basically random, it cannot be described with a deterministic model; its description must include a measure of its inherent randomness and thus uncertainty. For practical purposes, the required description may have to be limited to the main descriptors of interest, which are the central value (such as the mean or median) and its measure of dispersion (e.g. standard deviation or coefficient of variation). Available observational data can be used to estimate the central value and the degree of dispersion of the possible realizations.

Example

Suppose that the compressive strength of concrete used in a major structure is of interest. For purposes of illustration, assume that 15 cylinders were sampled from the concrete mixes used in the construction, and tested in compression with the following results:

6.5 ksi	6.1 ksi	4.7 ksi
4.3	4.8	5.7
5.2	5.5	5.2
5.8	4.2	4.1
5.0	5.1	6.3

On the basis of these observations, the sample mean and sample standard deviation are obtained as follows:

Sample Mean, = 5.23 ksi

Sample Standard Deviation, = 0.75 ksi

The corresponding coefficient of variation, therefore, is

$$= \frac{0.75}{5.23} = 0.14$$

The above sample mean, μ = 5.23 ksi, is of course an estimated value of the true mean strength of the concrete (which must remain unknown). The fact that there is significant scatter in the observed strengths of the different cylinders gives rise to uncertainty in the actual strength of any parts of the structure; this is the uncertainty due to the inherent randomness of concrete strengths, and is measured by the c.o.v. of δ = 0.14.

In addition to the uncertainty due to randomness as represented by the above c.o.v. of δ = 0.14, there may be additional uncertainty associated with errors in the estimation of the above mean value, as described below.

Uncertainty in prediction (model imperfections) -- A model (e.g. theoretical or empirical equation) or method used for predicting or estimating reality, will generally be imperfect; this is especially true of models used in engineering. As used here, a "model" is meant to be any technique or method for predicting or estimating the real-world condition. Such imperfections may lead to systematic error (i.e. bias) as well as dispersive error (random error) in the prediction. As observed earlier, if the underlying physical phenomenon is random, the prediction usually pertains to the central value of the underlying phenomenon.

For example, from a set of experimental data, the true mean-value may be estimated with the sample mean of the data, as illustrated above. Conceivably, if the same experiment were repeated and other sets of data were obtained, the sample mean estimated from each set of data would likely be different; the collection of all of the sample means will also have a mean-value, which may be different from the individual sample means, and a corresponding standard deviation. Conceptually, the mean-value of the sample means may be assumed to be the true mean-value. Then, the difference (or ratio) of the available sample mean to the mean sample mean is the systematic error or bias, whereas the c.o.v. or standard deviation of the several sample means is the dispersive error. Bias may be caused also by factors not accounted for in the model and that tend to consistently bias the estimate in one direction (or the other).

Referring again to the example discussed above, there is first of all an additional random sampling error which is given by

$$\Delta_1 = 0.14/\sqrt{15} = 0.04$$

The estimated mean concrete strength of 5.23 ksi may contain further error. For instance, the concrete cylinders may be cured under laboratory condition which would tend to raise the strength over that in the field; also, compaction and direction of casting may also reduce or influence the strength of field concrete, whereas confinement of the concrete will tend to increase its strength. It is, of course, difficult to evaluate or determine the effects of these extraneous factors on the strength of field concrete. Suppose that in the judgments of experts, the strength of laboratory concrete cylinders is 10% to 21% higher than the strength of field-poured specimens (Bloem, 1968). This information, therefore, suggests that the mean strength of field concrete will range between 10% and 21% lower than that of corresponding laboratory concrete cylinders. On this basis, and assuming a uniform probability distribution within this range (see Section 2.3.5), the systematic bias in the estimated mean concrete strength of μ = 5.23 ksi would be,

$$\text{mean bias factor, } \nu = \frac{1}{2}(0.79 + 0.90) = 0.85;$$

and the corresponding dispersive uncertainty in the estimated mean value, expressed in c.o.v., is

$$\Delta_2 = 0.58\left(\frac{0.90 - 0.79}{0.90 + 0.79}\right) = 0.04.$$

The total dispersive uncertainty in the estimated mean-value, therefore, is

$$\Delta = \sqrt{\Delta_1^2 + \Delta_2^2}$$

$$= \sqrt{0.04^2 + 0.04^2} = 0.06.$$

The concepts presented above are merely extensions or generalizations of the notion of estimation error that is well-established, for example in measurement theory (Parratt, 1961). In measurement theory, the estimated mean-value from a set of observations is usually used to represent the true measurement (state of nature); the error of the estimated mean-value consists of systematic and dispersive (or random) components. The systematic component may be due to certain well-identified factors whose effects can be determined (at least judgmentally) and thus can be corrected through a fixed or constant correction; whereas, the random component, called standard error in measurement theory, may be represented as a range of possible corrections which may be handled through statistical techniques. The systematic error, therefore, is a "bias" in the prediction or estimation, whereas the random error represents the degree of "dispersiveness" of the possible errors.

In general, therefore, the systematic error in prediction may be corrected by applying a constant bias correction factor; however, the dispersive error requires a statistical treatment and may be represented by the standard deviation (or coefficient of variation) of the predicted mean-value. In other words, the systematic bias is a recognizable fault in the model that will consistently underestimate or overestimate the state of nature, whereas the dispersive error is the statistical error in the estimated mean or median value. Objective determination of the bias, as well as of the dispersive error, would require repeated data on the sample mean (or median) values as discussed previously; data for these purposes, however, are invariably limited and hence must be augmented by judgments. The above discussions may be summarized as follows:

1. Through methods of prediction, we obtain (for a random phenomenon):

[illegible] = estimate of the mean-value

[illegible] = standard deviation (representing randomness)

2. An assessment of the accuracy or inaccuracy of the prediction, specifically with reference to [illegible], obtaining;

[illegible] = mean bias (or systematic error in the predicted [illegible])

[illegible] = dispersive error in [illegible].

In other words, prediction (in the case of a random phenomenon) usually yields an estimate of the mean or median value, [illegible], and associated c.o.v. [illegible]; whereas, error in the estimated mean-value may include a systematic component [illegible] and a dispersive component [illegible].

<u>Remarks</u> -- It is sometimes difficult to distinguish between randomness and the dispersive error of prediction. Indeed, all dispersive uncertainty may be the result of our inability to describe nature precisely. Randomness may be the result of having to use a relatively crude model; otherwise, if a more refined model were possible the degree of scatter in the observational data may be significantly reduced. For example, in determining the strength of a material, if the strength is described at the molecular level the measured scatter would be much less than that of the strength measured through an engineering specimen. Therefore, the uncertainty associated with randomness is really also due to imperfections in the material model. From this standpoint, a clear distinction between dispersive uncertainties due to randomness and model imperfections may become clouded. For certain purposes, such as the development of relationships for design, it is the total degree of uncertainty that is important, irrespective of its source and type; hence, any effort to distinctly separate these uncertainties into randomness and model imperfections, and to separately evaluate their effects, may sometimes be unnecessary or unimportant.

2.3.3 Measures of Uncertainty

The uncertainty due to inherent randomness requires statistical measure; normally this may be in terms of the standard deviation or the associated coefficient of variation. In terms of the coefficient of variation, the uncertainty due to randomness will be denoted by [illegible].

Uncertainty in prediction, of course, is associated with the errors in the prediction. Again, by model is meant any means or method for predicting or estimating the real condition. In practice, the prediction error may be limited to the error in the estimated central value (mean or median); the systematic error represents a consistent bias in the estimated mean or median value and therefore may be measured by a constant bias factor. The random error represents the dispersiveness of the estimated mean or median; this component may also be measured by the coefficient of variation, and will be denoted by Δ , in contrast to the randomness which is δ.

Whereas the systematic error can be accounted for by applying a constant bias factor to the estimated mean or median value, the uncertainty associated with randomness, δ, as well as model imperfections, Δ , requires statistical treatment. Such dispersive uncertainty may be expressed in terms of the standard deviation or the coefficient of variation as indicated above. However, other quantities may also be used for the same purpose. In particular, for strategic purposes in which the lognormal distribution is widely used, the standard deviation of the logarithm is also a convenient measure of the dispersive uncertainty. The relationships between these alternative uncertainty measures are as follows:

Denote, σ_X = standard deviation of X;

Ω_X = coefficient of variation of X;

ζ_X = standard deviation of $\ln$ X.

where $\ln$ stands for the natural logarithm.

Then

$$\Omega_X = \sigma_X/\mu_X,$$

where, μ_X is the mean-value, and (Ang and Tang, 1975)

$$\zeta_X^2 = \ln (1 + \Omega_X^2),$$

conversely,

$$\sigma_X = \Omega_X \mu_X$$

and,

$$\Omega_X = \sqrt{e^{\zeta^2} - 1}$$

It may be emphasized that whereas the standard deviation, σ_X, and the coefficient of variation, Ω_X, refer directly to the dispersion in the possible values of X, the measure ζ_X refers to the dispersion in the values of $\ln$ X. This means that either σ_X or Ω_X is a measure of the dispersion in X; whereas, strictly speaking, ζ_X is a measure of the dispersion in $\ln$ X.

2.3.4 Model for Uncertainty Analysis

The seemingly different types and sources of uncertainty discussed above may be analyzed with the following model.

Suppose that the true state of nature is X, whose actual realization is unknown (e.g., the true strength of concrete). Prediction or estimation of X, therefore is necessary; for this purpose, a predictive model, denoted $\hat{X}$, may be used. As $\hat{X}$ is a model of the real world, imperfections in the model can be expected; the resulting prediction, therefore, will contain error, and a correction N may be necessary. Then, the state of nature may be represented by (Ang, 1973)

$$X = N\hat{X} \tag{10}$$

If the state of nature X is inherently random, the model $\hat{X}$ should be a random variable. When data are available, for example, in the form of a set of sample observations $(x_1, x_2, \ldots, x_n)$, then the mean $\bar{x}$ and variance σ^2 of $\hat{X}$ may be estimated using standard statistical techniques; from which the coefficient-of-variation is $\delta = \frac{\sigma}{\bar{x}}$, representing the uncertainty due to randomness.

For generality, the correction N may also be considered to be a random variable, whose mean-value ν represents the mean correction for systematic error or bias in the predicted mean-value, $\bar{x}$, whereas its coefficient-of-variation, Δ, represents the dispersiveness in the possible error of the predicted mean-value $\bar{x}$ (i.e. the random error). In particular, Δ would include the random error in $\bar{x}$ due to sampling which is given by $\frac{\sigma}{\sqrt{n}}$, where n is the sample size of the available data. However, Δ as well as ν should include also the effects of factors not reflected in the data.

It is reasonable to assume that N and $\hat{X}$ are statistically independent; on this basis, the correct mean-value of X, following Eq. 10, is,

$$\mu_X = \nu\bar{x} \tag{11a}$$

Of course, if there is no bias in $\bar{x}$, then $\mu_X = \bar{x}$; moreover, once the bias is determined, the "correct" mean-value μ_X may be used.

The total c.o.v., representing the total dispersive uncertainty in the prediction of X then becomes,

$$\Omega_X \approx \sqrt{\delta_X^2 + \Delta_X^2} \tag{11b}$$

The above discussion, of course, pertains only to a single variable. Oftentimes the bias and uncertainty in a function are of interest. For example, if Y is a function of several variables $X_1, X_2, \ldots, X_n$, or

$$Y = g(X_1, X_2, \ldots, X_n) \tag{12}$$

in which $\mu_{X_i} = \nu_i\bar{x}_i$ and Ω_{X_i}, for i = 1, 2, 3, ..., n, have been determined, then the mean-value and uncertainty of Y are of concern.

In this case, an idealized (or model) function g could be used, and a correction N_g may be necessary, such that

$$Y = N_g\,\hat{g}(X_1,\ldots,X_n)$$

in which N_g has mean-value ν_g and c.o.v. Δ_g. On the basis of first-order approximations (Ang & Tang, 1975), the mean-value of Y is,

$$\mu_Y \simeq \nu_g \cdot \hat{g}(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}) \tag{13}$$

where ν_g is the bias in g(...), and

$$\Omega_Y^2 \simeq \Omega_g^2 + \frac{1}{\mu_Y^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}\, c_i c_j \sigma_{X_i} \sigma_{X_j} \tag{14}$$

in which:

ρ_{ij} = correlation coefficient between X_i and X_j; and

$c_i = \frac{\partial g}{\partial X_i}$, evaluated at $\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}$.

The above first-order approximate mean-value of the functional Y may be improved using a second-order approximation (Ang and Tang, 1975). In this case, we obtain an improved mean-value as follows (for uncorrelated $X_1, X_2, \ldots, X_n$)

$$\mu_Y \simeq \nu_g \cdot \hat{g}(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}) + \frac{1}{2} \sum_{i=1}^{n} \left(\frac{\partial^2 g}{\partial X_i^2}\right) \sigma_{X_i}^2 \tag{15}$$

whereas, if the variates are correlated,

$$\mu_Y = \nu_g \cdot \hat{g}(\mu_{X_1}, \ldots, \mu_{X_n}) + \frac{1}{2} \sum_i \sum_j \left(\frac{\partial^2 \hat{g}}{\partial X_i \partial X_j}\right) \rho_{ij}\, \sigma_{X_i} \sigma_{X_j} \tag{15a}$$

2.3.5 Estimation and Assessment of Uncertainty Measures

The implementation of the above model requires the assessment and quantitative estimation of the uncertainty measures associated with the individual variables $X_1, X_2, \ldots, X_n$. Clearly, the validity of a calculated probability will depend on the credibility of the uncertainty measures evaluated for each of the variables. Methods for these purposes will depend on the available data and information (or the lack thereof), as well as the form in which the available data are presented. Some of these methods are suggested below; however, they are not all-inclusive, and depending on the situation at hand, other methods may have to be devised as necessary.

When Sample Data are Available -- If a set of observational data is available; for example, a set of sample data $x_1, x_2, \ldots, x_n$. Using common statistical estimation techniques, the mean-value of the underlying random variable is obtained as

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

whereas, the corresponding variance would be

$$\sigma_X^2 = \frac{1}{n-1} \sum_{i=1} (x_i - \bar{x})^2$$

From which, the uncertainty associated with randomness (in this case) is given by the c.o.v.

$$\delta_X = \frac{\sigma_X}{\bar{x}}$$

The estimated mean-value may not be totally accurate relative to the true mean. The estimated mean-value given above is unbiased; however the dispersive error of the estimated mean-value $\bar{x}$, which is the standard deviation of $\bar{x}$, becomes (Ang and Tang, 1975);

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$$

Hence, the associated c.o.v. of $\bar{x}$ is,

$$\Delta_X = \frac{\sigma_{\bar{X}}}{\bar{x}}$$

It should be emphasized that the above dispersive uncertainty in $\bar{x}$ is limited only to the sampling error. In particular, the uncertainty in $\bar{x}$ due to other factors cannot be assessed from the available data, if the effects of these factors were not included in the tests from which the data were obtained.

When Range of Values is Known -- In engineering, the information that may be available is often in the form of the lower and upper limits of a variable. In such a case, the mean-value of the variable and the underlying uncertainty may be estimated on the basis of the given range and a prescribed distribution within this range. For example, for a variable X, if the lower and upper limits of its value are x_ℓ and x_u, the mean and coefficient-of-variation of X may be determined as follows:

Prescribing a uniform PDF between x_ℓ and x_u, the mean-value is then,

$$\bar{x} = \frac{1}{2} (x_\ell + x_u)$$

whereas the c.o.v. is (Yucemen, et al, 1973),

$$\delta_X = 0.58 \left(\frac{x_u - x_\ell}{x_u + x_\ell}\right)$$

Alternatively, if a symmetric triangular distribution is prescribed within the limits x_ℓ and x_u, the coefficient-of-variation becomes

$$\delta_X = 0.41 \left(\frac{x_u - x_\ell}{x_u + x_\ell}\right).$$

On the other hand, if the range corresponds to a given confidence bound, then the c.o.v. may be evaluated accordingly assuming an appropriate underlying distrubiton (e.g. normal or lognormal).

Purely on the basis of the data, the estimated mean-value $\bar{x}$ may not contain any bias (e.g. if an unbiased estimator is used). However, if there are important factors whose effects were not reflected in the data, then the effect of such factors may introduce bias into the estimated $\bar{x}$. The determination of such bias (i.e., ν) may often have to be based on engineering judgment.

Functionals -- The most common form of data or information that may be available for the statistical analysis of a functional is that from regression analysis. For example, in Fig. 6 is shown a hypothetical scattergram of Y as a function of x. From the scatter of the data, a regression equation (either linear or nonlinear as appropriate) may be developed on the basis of least squares error. The regression equation then gives the conditional mean-value of Y as a function of x, whereas the conditional standard deviation of Y for given x, $\sigma_{Y|x}$, represents the randomness about the regression equation; from which the conditional coefficient-of-variation is,

$$\delta_{Y|x} = \frac{\sigma_{Y|x}}{\bar{y}}$$

in which $\bar{y}$ is given by the regression equation. There may be bias in the regression equation; this could arise from factors that were neglected in developing the data of Fig. 6. The effect of these factors on the regression equation, therefore, represents the bias in the derived regression equation; determination of this bias again may have

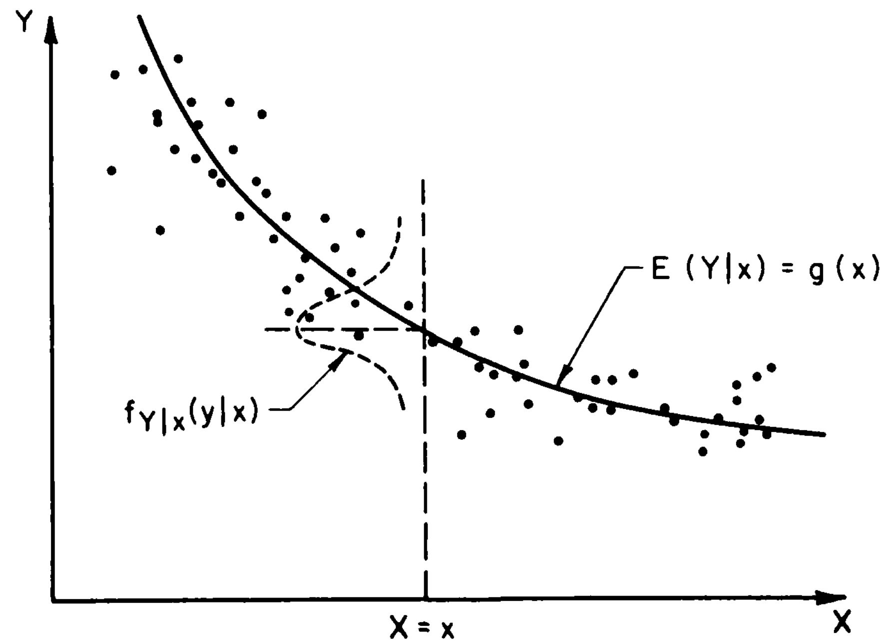


FIG. 6 REGRESSION OF Y ON X

to be based on subjective judgment, unless additional data can be developed to evaluate the effects of these factors on the regression equation. Such information, however, is seldom available.

Comments on Estimating Uncertainty -- It may be emphasized that although the dispersive uncertainties due to randomness and those associated with model imperfections may have to be assessed or estimated separately, the distinction between these two sources (or types) of uncertainty may not always be fruitful or necessary. In particular, both types of uncertainty will contribute to our inability for absolute assurance of survivability or kill potential; therefore, whether the uncertainty is due to randomness or to the dispersive error in prediction may not be that important. In problems of assessment, distinguishing between these two types of uncertainty may be more appropriate; however, in developing relationships for design or for decision-making, such a distinction would be unnecessary and serves only to introduce ambiguity.

The scatter (or dispersion) in most data from observations are the result of randomness, and therefore are appropriate for determining the uncertainty due to randomness. Seldom is there data available for assessing the uncertainty due to model imperfections; for this purpose, data showing the scatter in the estimated mean-values would be required. To obtain the scatter on the mean-value from measured data would entail multiple sets of observations; i.e., one mean-value from each set. Clearly, the data necessary for estimating the uncertainty due to model imperfections would be more extensive than those required for evaluating randomness.

Invariably, therefore, the uncertainty associated with the estimated mean-value (model imperfections) must be assessed largely on the basis of engineering judgment. This may take the form of a range of possible values for the true mean; given the range of the possible mean-values, the associated c.o.v. may be evaluated by assuming a probability density function (PDF) within the range.

The examples illustrated in Sect. 2.5 may serve to clarify some of the concepts and procedures described above.

2.4 Conditional and Total Probabilities

The notion of a conditional probability may be useful for special situations; this is particularly pertinent when considering problems in which certain key information is missing or when the uncertainty is very large. For example, when the mean-value is essentially unknown or when its estimate could potentially contain very large error; i.e. large c.o.v. Δ . Such situations are often encountered in target planning, in which the capability of a given target may have to be estimated with virtually no information. In these cases, the required probabilistic analysis (e.g. probability of kill) may have to be based on certain specific assumptions, and thus a conditional probability is appropriate; whereas if several assumptions (or conditions) are possible and their relative likelihoods may be postulated, then the total or expected probability may be useful.

The following examples may illustrate these concepts.

Example -- Suppose that the potential or conceivable site conditions of an enemy installation can be classified into one of three possibilities, which may be described as follows:

Site Condition A -- On soft soil

Site Condition B -- On medium stiff material

Site Condition C -- On hard rock

In planning a targeting strategy, information regarding the site condition of the enemy's installation obviously would be important. In this regard, the information available from intelligence could be in one of the following:

Case 1 -- The site condition is completely known; in this case, of course, the appropriate site condition should be considered and the other two possibilities discarded for the particular enemy installation.

Case 2 -- The ground condition of the site is only partially known; for example, intelligence might say that the site is most likely to be on hard rock, but could also be on medium or soft material. In addition suppose that in the judgment of the target analyst the relatively likelihoods of soft, medium, and hard ground are 7, 2, and 1.

Case 3 -- The condition of the site is completely unknown (absence of intelligence); however, the above three possible ground conditions are all-inclusive. In the absence of other information, the three possible ground conditions may be assumed to be equally likely for the site, or other relative likelihoods may be judgmentally prescribed.

Properly, each of the three cases stipulated above should be treated differently. Obviously the information on the site condition diminishes from Case 1 to Case 3, and this fact should be taken into consideration.

Suppose further that the probability of kill, p_K, corresponding to each of the three ground conditions at the site are, respectively, as follows (for a given weapon yield and CEP):

Site Condition	p_K
A	0.9
B	0.5
C	0.1

There are, of course, conditional probabilities; i.e. conditional on the site condition of the target.

Depending on the intelligence information, the probability of kill may be evaluated as follows:

If there is complete information (i.e. Case 1) regarding the site condition, the probability of kill would be

$$p_K = 0.9, \text{ or } 0.5, \text{ or } 0.1$$

depending on whether site condition A, B or C applies.

However, if there is partial intelligence information, as described in Case 2 above, the probability of kill would be calculated as,

$$p_K = \frac{7}{10} \times 0.9 + \frac{2}{10} \times 0.5 + \frac{1}{10} \times 0.1 = 0.74$$

Whereas, if there is no information (Case 3) on the ground condition of the enemy installation, i.e. absence of intelligence, the three possible site conditions may be assumed to be equally likely, and the corresponding probability of kill would be

$$p_K = 1/3\ (0.9 + 0.5 + 0.1) = 0.5$$

The probability appropriate in Case 1 above is a <u>conditional probability</u>; whereas in Cases 2 and 3, it is a <u>total or expected probability</u>.

<u>Special Observation</u> -- In Case 2 above, if the relative likelihoods between soft and hard ground were reversed; i.e. 1, 2, and 7 for soft, medium, and hard ground, the probability of kill with the same weapon system would be

$$p_K = \frac{1}{10} \times 0.90 + \frac{2}{10} \times 0.5 + \frac{7}{10} \times 0.1 = 0.26$$

Observe that this is less than the $p_K = 0.5$ of Case 3 (absence of intelligence), which may appear to suggest that "ignorance is bliss." However, if the ground condition is more likely to be hard than soft, the probability of kill should be closer to 0.1, and therefore the availability of the partial intelligence information simply points out that the weapon system would most likely be inadequate, whereas without the intelligence information the targeter may erroneously use the weapon believing that $p_K = 0.5$.

<u>Example</u> -- A similar situation could arise also from a strategic defense standpoint. The survivability of a structure or facility could depend on the weapon system used by the enemy; for example, an underground installation may be more vulnerable to a single large-yield weapon than to a number of small-yield weapons. That is, there are two possible enemy threats, namely threats A and B; the survival probability of the installation will depend on the threat. Suppose that these are as follows:

	Threat	p_S
(A)	Single large weapon	0.2
(b)	Several small weapons	0.9

If there is information on the threat, then the survival probability is either 0.2 or 0.9 depending on whether the enemy uses threat A or threat B. However, if intelligence information is not definite; only that there is a much higher likelihood of threat B than threat A, say by a factor of 3 to 1, then the survivability would be

$$p_S = 1/4 \times 0.2 + 3/4 \times 0.9 = 0.73$$

whereas, if there is no information on the enemy threat (absence of intelligence) the two threats may be assumed to be equally likely; in which case, the survival probability would be

$$p_S = 1/2\ (0.9 + 0.2) = 0.55$$

The above examples should serve to emphasize that when certain information is available, it defines the appropriate conditional probability; whereas when there is partial information or no information (as in the case of the ground condition of the enemy installation), the expected probability may be appropriate.

Remarks -- In problems where the probability of survival, or probability of kill, depends signficantly on certain conditions, which are largely unknown, the only information that can be developed is in terms of conditional probability. Such conditional probabilities may then be used by the targeteer or survivability analyst in conjunction with available intelligence information.

2.5 Illustrative Examples

2.5.1 Example (Reliability of Long Columns)

In order to illustrate some of the main concepts described and presented above, a pedagogical example involving the structural safety of long steel columns is considered. This is a hypothetical example of a problem familiar to all structural engineers, and is developed expressly for the purpose of illustrating the probability and statistical concepts and procedures described earlier. The assumptions that are necessary are reasonably realistic; nevertheless, they are made merely for purposes of illustration.

For reasonably long steel columns, in which failure will most likely be caused by elastic buckling, the strength of a column may be predicted with the Euler formula, which gives the critical buckling stress as

$$f_{cr} = \frac{\pi^2 E}{(k \cdot \frac{L}{r})^2} \qquad (16)$$

in which:

- E = modulus of elasticity;
- L = column length;
- r = radius of gyration about the weak axis of the cross section;
- k = effective length coefficient;
  - $k = 1$, for columns with hinged-hinged end supports,
  - $k = 1/2$, for columns with fixed-fixed end supports.

Eq. 16 is based on certain idealized assumptions, including the following:

(i) the column is perfectly straight;

(ii) the material is linearly elastic;

(iii) the compressive load is applied axially, i.e. with no eccentricity.

Therefore, imperfections in the use of the Euler formula for predicting the buckling strength of actual columns can be expected, as one or more of the above idealizations may be violated in practice.

In the case of structural steel, the modulus of elasticity, E, is fairly uniform. Its mean-value is generally around 29,000 ksi, with a small coefficient-of-variation. On this basis, it is reasonable to assume $\nu_E = 1.0$; hence,

$$\mu_E = 29{,}000 \text{ ksi}$$

$$\delta_E = 0.03$$

The variability in the radius of gyration, r, would be the result of the variabilities in the cross sectional dimensions of the column; again, the associated coefficient of variation would be small, say $\delta_r = 0.05$, and $\nu_r = 1.0$.

In the case of E and r, the estimated mean or median values would be fairly accurate and, therefore, any error in the estimated mean values will be negligible; thus,

$$\Omega_E = \delta_E$$

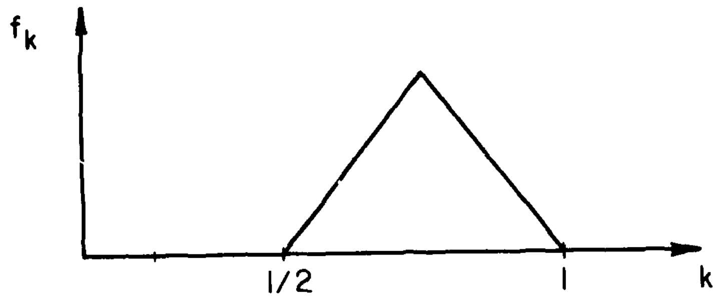
and,

$$\Omega_r = \delta_r.$$

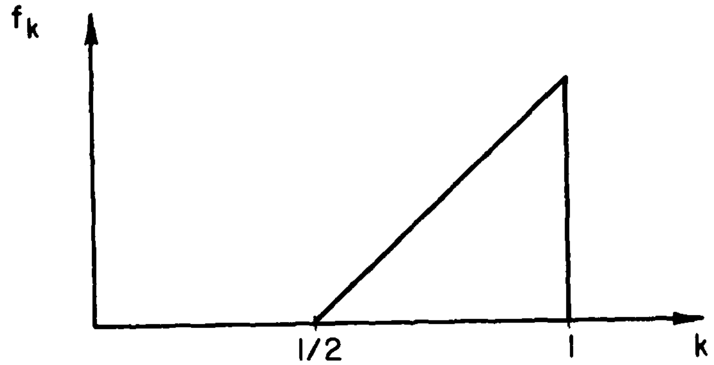
The end support conditions of a column directly affect its critical buckling stress; this is reflected in the factor k of Eq. 16. In an actual column, the end conditions would generally be between the hinged-hinged (k = 1) and the fixed-fixed (k = 1/2) conditions. Unless more definite information is available, the uncertainty associated with the effects of the end conditions for a column may be evaluated by assuming a simple PDF between the above two extreme end conditions. Furthermore, it is probably reasonable to assume that, in general, it is more likely for the end conditions to lie midway between these two extremes. Thus, a symmetric triangular PDF may be prescribed between the two extreme conditions, as shown in the figure below. On the basis of the assumptions prescribed above, the mean and coefficient-of-variation of k becomes (see Yucemen, et al, 1973):

$$\bar{k} = 0.75$$

$$\Omega_k = 0.41 \left(\frac{1 - 0.5}{1 + 0.5}\right) \approx 0.14$$



Alternatively, if it is the judgment that the actual conditions will tend more toward one extreme or the other, then a nonsymmetric triangular PDF may be more appropriate. For example, if it is believed that the actual end conditions are closer to the hinged-hinged supports, then the triangular distribution shown below is appropriate (Yucemen, et al, 1973)



In such a case, the mean and coefficient-of-variation of k would be;

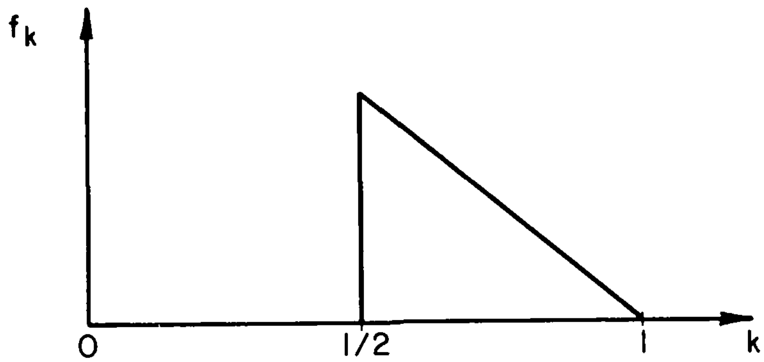
$$\bar{k} = 0.5 + 2/3 \times 0.5 = 0.833$$

$$\Omega_k = 0.707 \left(\frac{1 - 0.5}{0.5 + 2 \times 1}\right) = 0.14$$

whereas, if the actual end supports are believed to be closer to the fixed-fixed condition, then the following triangular distribution should be perscribed. In this case, the corresponding mean and coefficient-of-variation of k would be (Yucemen, et al, 1973):

$$\bar{k} = 0.5 + 1/3 \times 0.5 = 0.667$$

$$\Omega_k = 0.707 \left(\frac{1 - 0.5}{2 \times 0.5 + 1}\right) = 0.18$$



Finally, there may be bias and dispersive error in the Euler equation itself. In particular, the imperfection of the Euler equation may arise from the initial crookedness of a real column, as well as from any inelasticity of the material; both of which would violate the idealized conditions on which the Euler equation was based. To evaluate such bias and dispersive error of the Euler equation, test data that can be used to evaluate the accuracy of the equation would be required; otherwise, the necessary evaluations have to be based on engineering judgments.

For example, suppose laboratory test results of axially loaded steel columns with hinged-hinged conditions were found for columns with several $\frac{L}{r}$ ratios; the measured critical buckling stresses were reported as follows:

$\frac{L}{r}$	Measured f_{cr}
120	18.0 ksi
120	19.0 ksi
120	21.0 ksi
150	11.0 ksi
150	12.5 ksi
150	11.5 ksi
175	9.0 ksi
175	8.5

These data may then be used to evaluate the inaccuracy (i.e. bias and dispersive error) of the Euler equation, by comparing the measured f_{cr} to the theoretical f_{cr} (as given by the Euler equation). For this purpose, we evaluate using the above data as follows:

$\frac{L}{r}$	Measured f_{cr}	Mean Meas. f_{cr}	Theoretical f_{cr}	$\frac{\text{Mean Meas. } f_{cr}}{\text{Theo. } f_{cr}}$
120	18.0			
120	19.0	19.33	20.56	0.94
120	21.0			
150	11.0			
150	12.5	11.67	13.16	0.88
150	11.5			
175	9.0	8.75	9.67	0.91
175	8.5			

Assuming that the experimental results represent reality, the ratio of the mean measured f_{cr} to the theoretical f_{cr}, therefore, is a measure of the accuracy or inaccuracy of the model (i.e. in this case the Euler equation). In this regard, the mean-value of this ratio is 0.91 and the corresponding c.o.v. is 0.03. Therefore, there is a bias in the Euler equation; i.e. it generally over predicts the true buckling stress. According to the above (hypothetical) reported data, the buckling strength of long columns predicted with the Euler equation should be corrected by the bias factor of 0.91. Moreover, the error in the Euler equation also has a c.o.v. of 0.03. Hence, in this case, we have (with reference to Eqs. 13 and 14)

$$\nu_g = 0.91$$

$$\Omega_g = 0.03$$

On the basis of the above analyses, the mean-value and coefficient-of-variation of the buckling strength of long steel columns, therefore, are as follows (assuming a symmetric triangular PDF for k): By first-order approximation,

$$\mu_{f_{cr}} \simeq 0.91 \cdot \frac{\pi^2 \times 29{,}000}{(0.75\frac{L}{r})^2}$$

and

$$\Omega_{f_{cr}} \simeq \sqrt{\Omega_E^2 + 4\Omega_k^2 + 4\Omega_r^2 + \Omega_g^2}$$

$$= \sqrt{0.03^2 + 4(0.14)^2 + 4(0.05)^2 + 0.03^2}$$

$$= 0.30$$

All the above analyses pertain only to the resistance of columns. For the purpose of evaluating the probability of survival or failure of long steel columns, a similar analysis would be required also of the applied loading. For the present illustration, suppose that an analysis of the loading yields a total c.o.v. in the applied load of $\Omega_S = 0.45$, and $\nu_S = 1.0$ (i.e. no bias in the predicted mean load).

Also, for the present illustration, suppose that steel columns are designed with the following allowable stresses (i.e. code provision):

$$f_{all} = 15\left(\frac{125}{L/r}\right)^2; \qquad \text{for } \frac{L}{r} \geq 125$$

Columns proportioned with the above allowable stresses are implicitly designed with an underlying acceptable failure probability. Prescribing the lognormal distribution for the buckling strength as well as for the applied load, the failure probability is determined as follows:

The bias and uncertainty for all $\frac{L}{r}$ ratios are assumed to be the same. Then, the failure probability is,

$$p_F = 1 - \Phi\left(\frac{\ln \tilde{\theta}}{\zeta}\right)$$

where,

$$\tilde{\theta} = \bar{\theta}\sqrt{\frac{1+\Omega_S^2}{1+\Omega_{f_{cr}}^2}}, \quad \text{and} \quad \zeta = \sqrt{\zeta_{f_{cr}}^2 + \zeta_S^2}$$

in which: $\tilde{\theta}$ is the median safety factor, whereas $\bar{\theta}$ is the mean safety factor;

and,

$$\zeta_S^2 = \ln(1+\Omega_S^2)$$

$$\zeta_{f_{cr}}^2 = \ln(1+\Omega_{f_{cr}}^2).$$

In designing a column, the required area is determined from,

$$A \geq \frac{\bar{S}}{f_{all}}$$

Thus, the mean buckling capacity of the column is

$$\bar{R} \geq A \cdot \mu_{f_{cr}}$$

Therefore, the mean safety factor underlying columns designed with the above allowable stress is,

$$\bar{\theta} = \frac{\bar{R}}{\bar{S}} = \frac{A \cdot \mu_{f_{cr}}}{A \cdot f_{all}} = \frac{\mu_{f_{cr}}}{f_{all}}$$

$$= \frac{0.91 \cdot \dfrac{\pi^2 (29{,}000)}{(0.75\ L/r)^2}}{15\ \left(\dfrac{125}{L/r}\right)^2}$$

In this case, the mean safety factor for all L/r is,

$$\bar{\theta} = 1.98$$

Hence, the corresponding median safety factor is,

$$\tilde{\theta} = 1.98 \sqrt{\frac{1+0.45^2}{1+0.30^2}} = 2.08$$

and,

$$\zeta^2_{f_{cr}} = \ell n\ (1+0.30^2) = 0.086$$

$$\zeta^2_S = \ell n\ (1+0.45^2) = 0.184$$

Thus, the underlying failure probability is;

$$p_F = 1-\Phi\left(\frac{\ell n\ 2.08}{\sqrt{0.086+0.184}}\right)$$

$$= 1 - \Phi(1.41)$$

$$= 1 - 0.921$$

$$= 0.079$$

The above calculation is based on the first-order approximation. The calculated p_F can be improved by using the second-order approximation for the mean column strength; i.e. using Eq. 15. This would yield the following for the specific value of L/r = 150:

Eq. 15 yields,

$$\mu_{f_{cr}} = 0.91 \quad \frac{\pi^2 E}{(k \cdot \frac{L}{r})^2} + \frac{1}{2}\left[\frac{6\pi^2 E}{k^2 (L/r)^4} \cdot \sigma^2_{L/r} + \frac{6\pi^2 E}{k^4 (L/r)^2} \cdot \sigma_k^2\right]$$

$$= 0.91 \cdot \frac{\pi^2(29{,}000)}{(0.75\text{x}150)^2} + 3\left[\frac{\pi^2(29{,}000)}{(.75)^2(150)^4}(0.05\text{x}150)^2\right.$$

$$\left. + \frac{\pi^2(29{,}000)}{(.75)^4(150)^2}(0.14\text{x}0.75)^2\right]$$

$$= 20.58 + 3(0.057 + 0.443)$$

$$= 20.58 + 1.50$$

$$= 22.08 \text{ ksi}$$

For $L/r = 150$, the allowable stress is

$$f_{all} = 15(\frac{125}{150})^2 = 10.42 \text{ ksi.}$$

The mean safety factor, therefore, becomes

$$\bar{\theta} = \frac{22.08}{10.42} = 2.12$$

and,

$$\tilde{\theta} = 2.12\sqrt{\frac{1+.45^2}{1+.30^2}} = 2.23$$

The improved (2nd-order) failure probability then is

$$p_F = 1 - \Phi(\frac{\ell n\ 2.23}{\sqrt{.086+.184}}) = 1 - \Phi(1.54) = 1\text{-}0.938$$

$$= 0.062$$

2.5.2 <u>Example (Analysis of Test Data)</u>

This next example should serve to illustrate the analysis of available weapons effect test data; specifically pertaining to the evaluation of the uncertainty in the prediction of ground motions with range (or scaled range).

Field data for ground motions from nuclear tests have been analyzed and reported by a number of authors; e.g. Cooper (1973) and Perret and Bass (1975). Such data are invariably presented graphically in logarithmic plots of free-field ground motions with slant range, or scaled range $R/W^{1/3}$.

Test data are usually reported for earth materials that are classified generically as alluvium, tuff, and hard rock. The pertinent material properties, such as seismic velocity and density, may vary significantly within each type of material; however, there is not much to be gained by further subdivision of the material than the two or three generic types commonly used (Cooper, 1973).

It may be pointed out that within each material type, the scatter of the data represents the uncertainty associated with randomness only. In particular, these data cannot be used to evaluate the bias and uncertainty due to model imperfections.

All the field data reported by Cooper (1973) and by Perret and Bass (1975) were measurements from fully contained nuclear burst. In order to use the attenuation relations derived from contained burst for surface-contact or shallow-buried burst condition, Cooper (1973) suggested the use of a coupling factor, K; specifically, K = 0.16 for shallow-buried burst and K = 0.04 for surface-contact burst were recommended.

Data Analysis of Cooper (1973) -- Data from underground tests in granite and other hard rocks were analyzed by Cooper (1973); particle velocity and displacement data from contained bursts were presented as shown in Figs. 7 and 8. In Fig. 7 the compressive stress scale is also shown; this is based on the stress-velocity relation $\sigma_{KBAR} = \rho c v_{FPS}$, where ρ and c are the mass density and compressional wave speed, respectively, for granite.

The logarithmic mean lines through the data in Figs. 7 and 8 give the following attenuation equations for stress and ground motions:

$$\sigma_{KBAR} = 7\ W_{MT}^{2/3}\ R_{KFT}^{-2} \tag{17}$$

$$v_{FPS} = 200\ W_{MT}^{2/3}\ R_{KFT}^{-2} \tag{18}$$

$$d_{IN} = 140\ W_{MT}^{5/6}\ R_{KFT}^{-3/2} \tag{19}$$

Also, the lower and upper lines, in Figs. 7 and 8, bounding the data are given as follows:

$$3.5\ W_{MT}^{2/3}\ R_{KFT}^{-2} \le \sigma_{KBAR} \le 14\ W_{MT}^{2/3}\ R_{KFT}^{-2} \tag{20}$$

$$100\ W_{MT}^{2/3}\ R_{KFT}^{-2} \le v_{FPS} \le 400\ W_{MT}^{2/3}\ R_{KFT}^{-2} \tag{21}$$

$$70\ W_{MT}^{5/6}\ R_{KFT}^{-3/2} \le d_{IN} \le 280\ W_{MT}^{5/6}\ R_{KFT}^{-3/2} \tag{22}$$

The results obtained from Eqs. 17 through 19 are the median stress, velocity, and displacement. From Eqs. 20 through 22, the coefficients-of-variation associated with the data scatter may be evaluated. For example, for the particle velocity, assuming that the bounding lines correspond to the 90% confidence bounds, the standard deviation of the natural logarithm of the particle velocity is

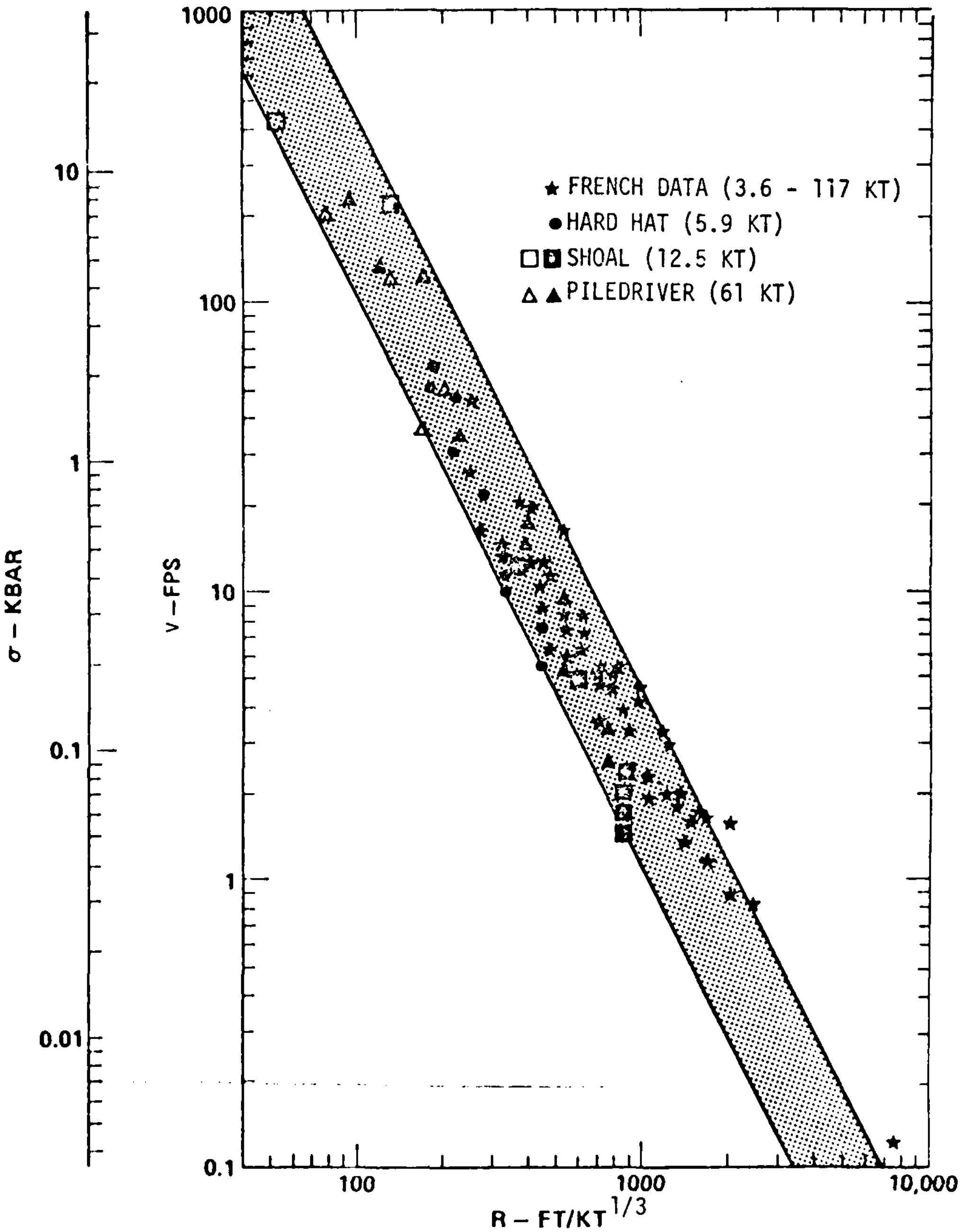


Fig. 7 Scaled Peak Particle Velocity and Stress from Tamped Nuclear Explosions in Granite (Open symbols were peak stress measurements.) -- After Cooper (1973)

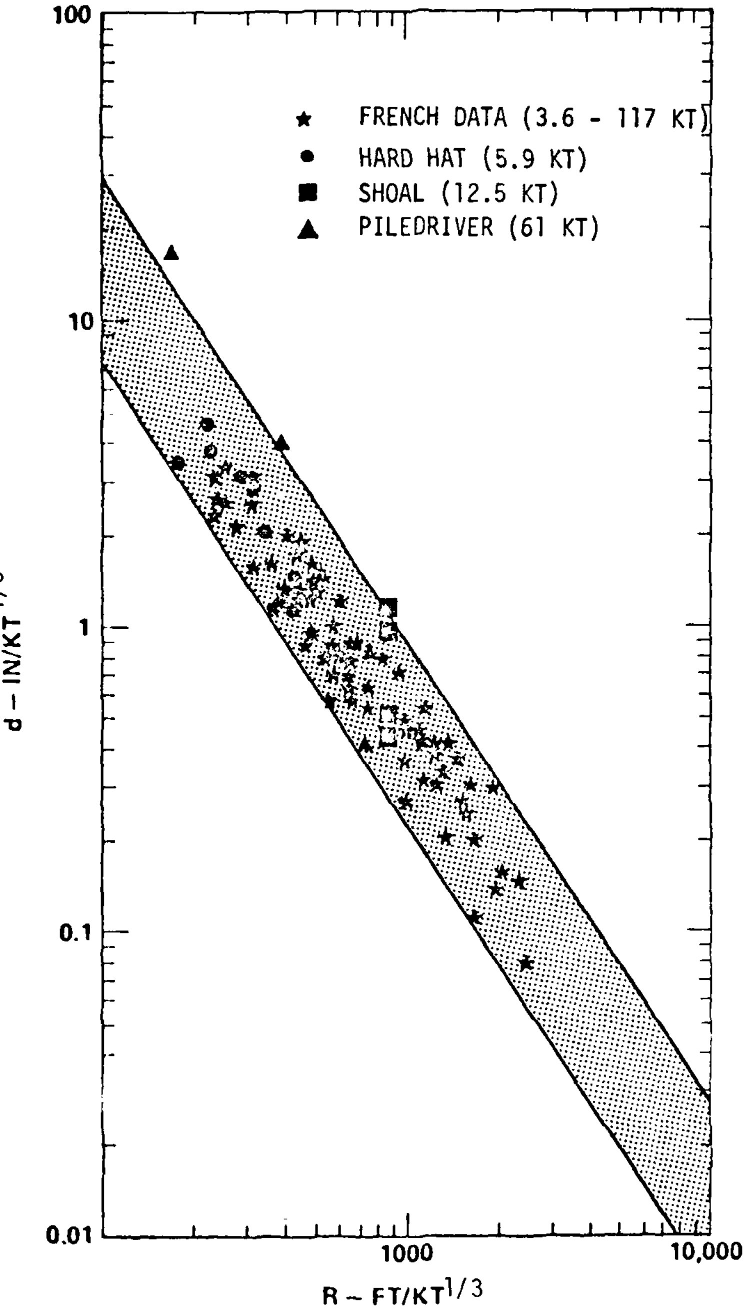


Fig. 8 Scaled Displacements from Contained Nuclear Explosions in Granite -- After Cooper, 1973

$$\zeta_v = \frac{\ell n\ 400 - \ell n\ 100}{2 \times 1.65} \approx 0.42$$

Thus, the c.o.v. of v is (see Sect. 2.3.3),

$$\delta_v = \sqrt{e^{0.42^2} - 1} = 0.44$$

Whereas, for displacement, the standard deviation of $\ell n\, d$ is

$$\zeta_d = \frac{\ell n\ 280 - \ell n\ 70}{2 \times 1.65} = 0.42$$

and the corresponding c.o.v. is also

$$\delta_d = 0.44$$

The c.o.v. of the stress, of course, is the same as that of the particle velocity.

Data Analysis of Perret and Bass (1975) -- Nuclear test data were analyzed also by Perret and Bass (1975). Statistical and regression analysis of the available data were performed; results were presented in logarithmic plots of peak motions with scaled range, as shown in Figs. 9 and 10. Aside from presenting the logarithmic mean (i.e. median) ground motions, values of the "variance factors" representing the degree of scatter of the data about the regression equations were also presented.

The regression equations giving the median ground motions were given in the following form:

$$x = C(KW)^a\ R^{-b} \tag{23}$$

in which x is the acceleration, velocity, or displacement; a, b and C are coefficients; W is the weapon yield in kiloton; K is the coupling factor; and R is the range in meters. Specific values of the coefficients C, a and b are given as shown in Table 1 below, for each type of material, as well as the appropriate range of applicability. In addition, the variance factor in each case are also given.

As defined by Perret and Bass, the variance factor is the exponential of the standard deviation of $\ell n C$. Hence, its natural logarithm yields the parameter ζ; i.e.

$$\zeta_C = \ell n(\text{variance factor})$$

On the basis of Eq. 23 and the coefficients given in Table 1 the equation for determining the pertinent median ground motion as a function of the slant range is obtained. For example, in wet tuff, the median particle velocity is given by

$$u = 6.61 \times 10^3\ (KW)^{0.52}\ R^{-1.56}$$

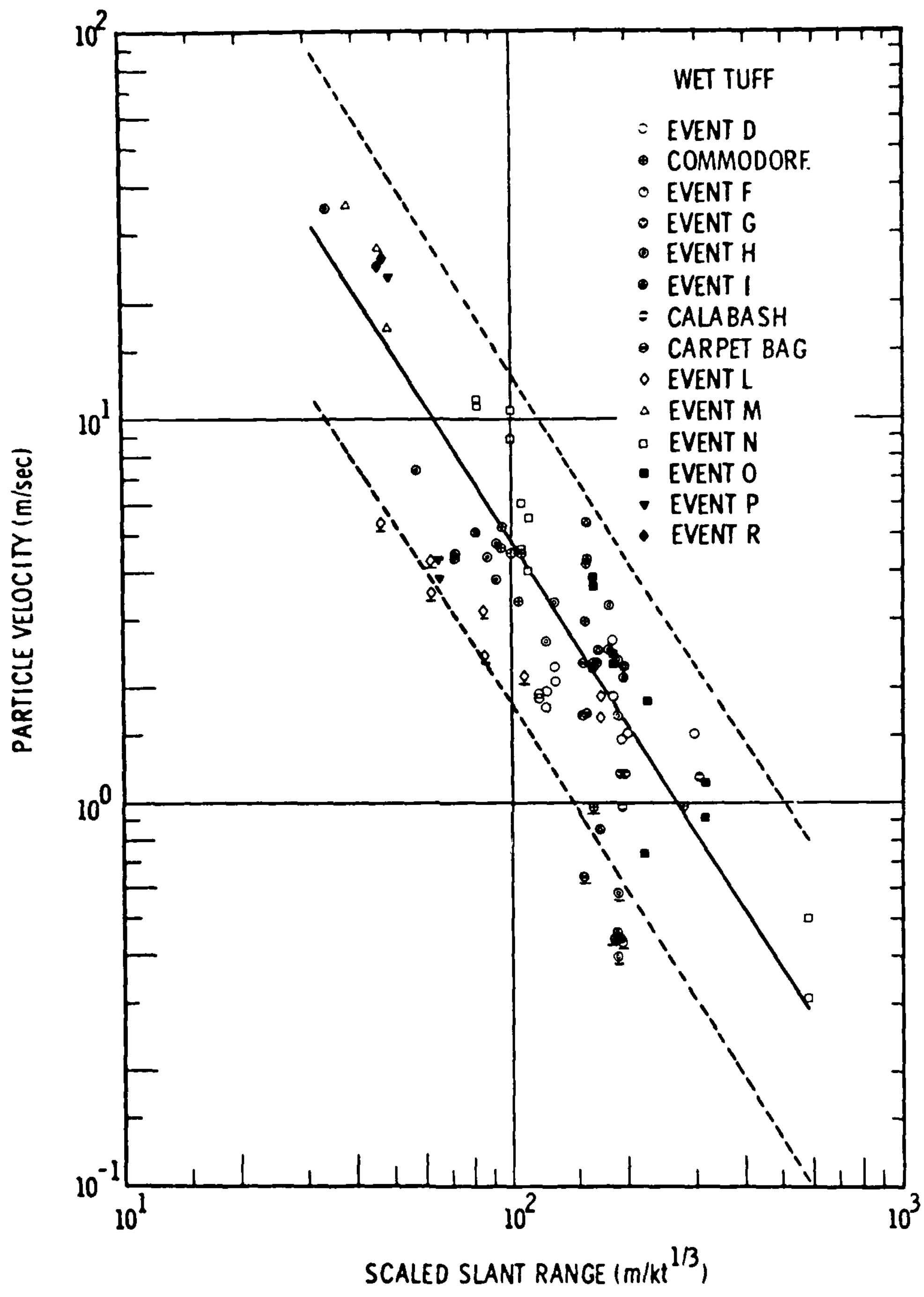


Fig. 9 Attenuation of particle velocity-wet tuff--
After Perret and Bass (1975)

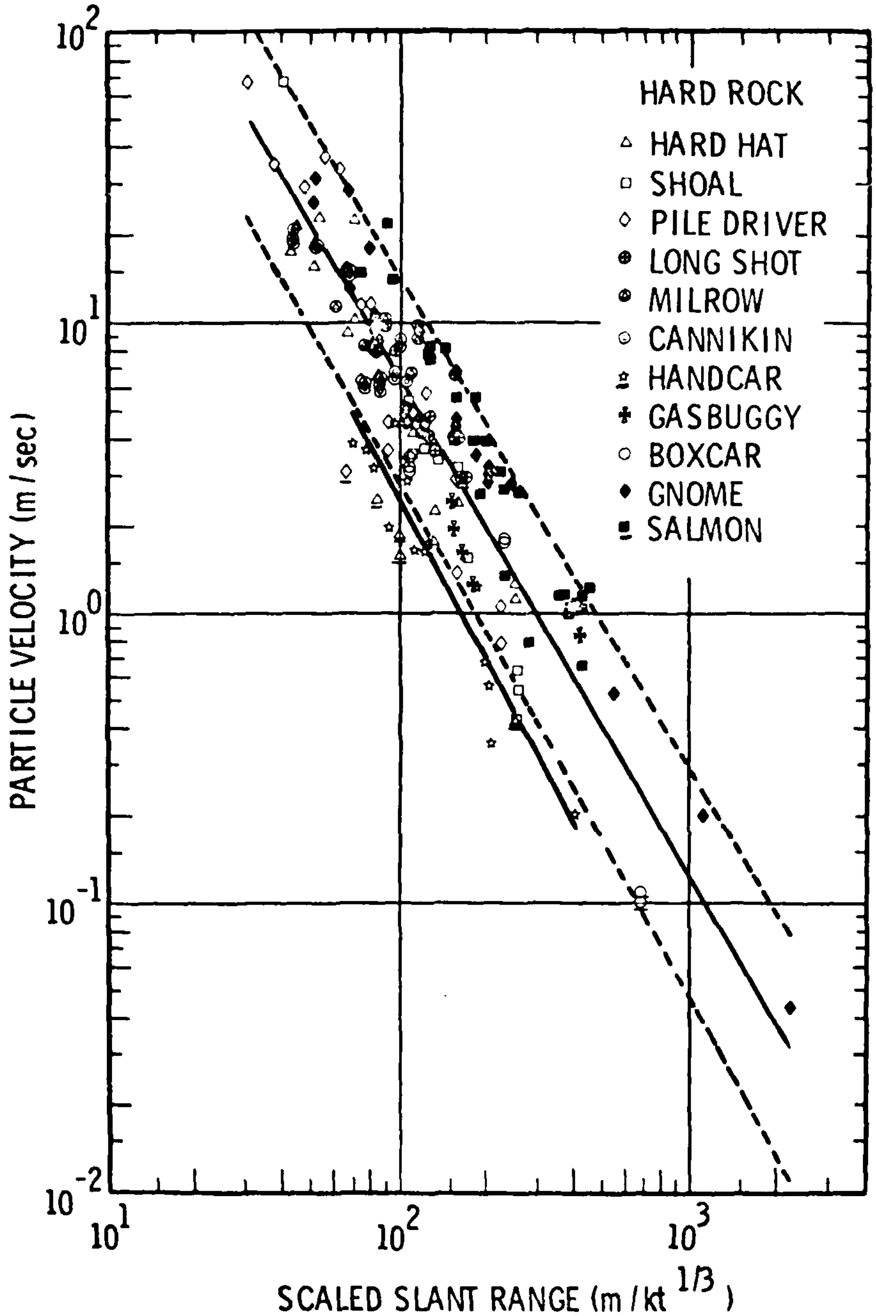


Fig. 10 Attenuation of particle velocity-hard rock--
After Perret and Bass (1975)

Table 1: Coefficients for Eq. 23 (after Perret and Bass, 1975)

Motion	Material	C	a	b	Applicable range, $m/kt^{1/3}$	Variance Factor
$a \cdot W^{1/3}$	Alluvium	2.24×10^{11}	1.93	5.78	20 - 80	6.39
$a \cdot W^{1/3}$	"	4.79×10^{4}	0.71	2.13	60 -350	2.34
u	"	1.52×10^{6}	1.09	3.27	30 -150	1.96
u	"	3.86×10^{1}	0.39	1.16	100 -350	2.09
$d/W^{1/3}$	"	3.44×10^{6}	1.01	3.04	40 -150	2.42
$d/W^{1/3}$	"	2.22×10^{2}	0.37	1.11	100 -350	1.72
$a \cdot W^{1/3}$	Dry Tuff	4.90×10^{10}	1.59	4.77	40 -150	4.33
$a \cdot W^{1/3}$	"	7.71×10^{4}	0.64	1.92	100 -500	2.12
u	"	1.85×10^{4}	0.66	1.98	40 -500	1.81
$d/W^{1/3}$	"	3.80×10^{5}	0.73	2.20	100 -500	3.11
$a \cdot W^{1/3}$	Wet Tuff	4.31×10^{7}	0.87	2.62	30 -600	2.21
u	"	6.61×10^{3}	0.52	1.56	30 -600	1.56
$d/W^{1/3}$	"	4.90×10^{6}	0.88	2.63	50 -600	2.63
$a \cdot W^{1/3}$	Hard Rock	9.29×10^{6}	0.77	2.32	90-2200	1.56
u	"	1.81×10^{4}	0.57	1.72	40-2200	1.39
$d/W^{1/3}$	"	8.72×10^{4}	0.63	1.88	70-2200	2.08

whereas, the corresponding equation for acceleration would be,

$$a \cdot W^{1/3} = 4.31 \times 10^{7} \, (KW)^{0.87} \, R^{-2.62}$$

Also, on the basis of the variance factors given in Table 1, the coefficient-of variation representing the uncertainty due to randomness in the measured data for wet tuff may be obtained as follows;

In wet tuff, the variance factor for particle velocity u is 1.56 .
Therefore,

$$\zeta_C = \ln 1.56 = 0.445$$

Hence, the c.o.v. is

$$\delta_C = \sqrt{e^{.445^2} - 1} = 0.468$$

It may be emphasized that the test data described above can be used only to evaluate the uncertainty due to randomness; i.e. the scatter in the measured data respresents or is caused by factors that are inherently random, including the natural heterogeneity of the soil and rock deposits. These data, however, do not provide a basis for evaluating the presence of systematic bias or dispersive errors in the models (in this case, the attenuation equations). In the present case, the bias refers to any tendency of the regression equations to over- or under-estimate the median ground motions; whereas the statistical uncertainty refers to the conceivable dispersion of the predicted median ground motion. The imperfections of the model would include the effects of factors that were not explicitly reflected in the field data, or in the attenuation equations. For these reasons, those factors that were not included in the experimental setup of the weapons tests should be identified. The effects of such factors on the ground motions, therefore, will contribute to the systematic and dispersive imperfections of the attenuation equations; these effects, however, may have to be evaluated largely on the basis of subjective judgments. The pertinent factors and their potential effects on ground motions should be carefully examined and assessed. Such examination and evaluation would require expertise in nuclear weapons effects, and ought to be performed by or with the cooperation of such experts. In this regard, the bases for all uncertainty measures should be explicitly documented and explained; where judgments are required, their bases should also be described. It is important that the measures of uncertainty used in a probabilistic analysis be credible. Credibility, however, is not enhanced when numbers (representing uncertainty measures) are simply given without apparent explanation or justification. Even though prediction errors may often have to be assessed subjectively through engineering judgment, they should be carefully and systematically analyzed and quantified; in particular, the basis for any judgment ought to be explained and carefully documented. Otherwise, it may appear to be arbitrary.

III. IMPLEMENTATION IN SURVIVABILITY/VULNERABILITY ANALYSIS AND DESIGN

3.1 Introductory Remarks

In implementing probability for strategic purposes, it is important to recognize that probability is necessary only to the extent that it is a quantitative measure of survivability or vulnerability. It cannot be overemphasized that, because of uncertainty, absolute survivability or vulnerability is not realistically possible; it may be assured only in terms of probability. However, the implementation of probability for this purpose does not necessarily mean that probabilistic analysis must be performed in all phases of evaluation and design; indeed, it is needed only as a tool for the analysis of uncertainty and its effect on survival probability. In many cases, those phases of engineering analysis that require probabilistic or statistical methods can be isolated and processed once, in such a form that the remaining steps in the actual design for survivability, or target planning for kill potential, can be carried out in conventional (deterministic) terms.

In other words, the implementation of probability in survivability and vulnerability problems should not necessarily change the conventional (usually deterministic) procedure. The use of probabilistic methodologies can be limited to the development of certain generic relationships that are necessary as a consequence of unavoidable uncertainties. In this sense, the real role of probability in engineering is that of supplementing existing deterministic methods of analysis and design; recognition of this role can serve to facilitate the effective and sensible implementation of probabilistic concepts and methods. In other words, probability and probabilistic methods are most effective if used to supplement that which is lacking in purely deterministic approaches, which is to provide quantitative means for uncertainty assessment and analysis of its effects on the assurance (or degree of assurance) of survivability and weapon effectiveness.

Implementation should take into account in particular the following:

(i) the objective of a probabilistic analysis;

(ii) the state and quality of available information;

(iii) the level of accuracy of a calculational method that is commensurate with the quality of available information.

If the purpose of a calculated probability is to estimate or assess the true probability of survival of a given system, the required probability may be limited to the consequence of the uncertainty due to randomness; this may be accompanied by an error bound on the estimate representing the uncertainty due to prediction errors. However, if the objective is to develop a design or to develop relationships useful for formulating designs, then the consequences of all sources of uncertainty, regardless of whether they are due to randomness or arising from prediction error, should be reflected in the calculated probability of survival; in this latter case, the two types of uncertainty, therefore, may be combined and analyzed together.

That is, if the objective is the evaluation of the probability of survival of a given structure or facility to a specified enemy attack, the probability of survival may be expressed in terms of an interval estimate of the true survival probability; any value within this interval represents the effect of randomness, whereas the range of the interval represents the effects of uncertainty associated with prediction errors.

On the otherhand, if the objective is the formulation of a design to withstand an enemy attack with a desired survival probability, then the effects of all sources of uncertainty must be included. For this purpose, the total uncertainty irrespective of whether it is due to randomness or arises from prediction error, is pertinent.

The emphasis in the present report is on the application of probability concepts for the formulation of structural design for survivability. Accordingly, the emphasis is principally limited to the elucidation of the expected probability approach. For purposes of design or decision making, a calculated probability of survival or kill should preferably be unambiguous. Ambiguity can be avoided by using the expected probability approach, as this gives a point estimate of the probability of survival. On the otherhand, an interval estimate of the probability of survival would be ambiguous as there is no single value representing the true probability of survival.

3.2 Review of Basic Approaches

3.2.1 The Expected Probability Approach

In Eq. 4 or 5, if the c.o.v. Ω, or standard deviation σ, includes all the dispersive uncertainties in R and S, the resulting probability of failure or survival is effectively a "total probability," or expected probability, in the sense that the consequences of all sources of uncertainty (associated with randomness as well as with errors of prediction) are reflected in the calculated probability. That is, in this case, the mean (or median) value of the resistance is

$$\mu_R = \nu_R \bar{r}$$

whereas the coefficient-of-variation of the resistance is (based on first-order approximation)

$$\Omega_R \simeq \sqrt{\delta_R^2 + \Delta_R^2}$$

Of course, if there is no bias in the prediction model, $\nu_R = 1.0$ and $\mu_R = \bar{r}$.

Similarly, the corresponding mean (or median) and coefficient-of-variation of the applied weapon effect are

$$\mu_S = \nu_S \bar{s}$$

and,

$$\Omega_S \simeq \sqrt{\delta_S^2 + \Delta_S^2}$$

where;

ν_R, ν_S = the mean bias factors in $\bar{r}$ and $\bar{s}$;

δ_R, δ_S = c.o.v. representing the uncertainties due to randomness;

Δ_R, Δ_S = c.o.v. representing the dispersive errors in the predicted mean-values $\bar{r}$ and $\bar{s}$.

Observe that any bias in the calculated r and $\bar{s}$ are corrected with the deterministic bias factors ν_R and ν_S; such biases, therefore, are implicitly reflected in the calculated probability of Eq. 4 or 5.

The resistance R and weapon effect S are often functions of the respective variables; that is,

$$R = g_1(R_1, R_2, \ldots, R_n) \tag{24}$$

and,

$$S = g_2(S_1, S_2, \ldots, S_m) \tag{25}$$

In these cases, the total uncertainty in each variable, R_i or S_j, may be evaluated individually; e.g. $\Omega_i = \sqrt{\delta_i^2 + \Delta_i^2}$, and then the total uncertainty in R and S determined through Eq. 14. This assumes that the uncertainties due to randomness and prediction errors between variables are statistically independent or uncorrelated. However, if the randomness between two variables, e.g. R_i and R_j, are statistically independent, whereas the corresponding model imperfections are correlated (or vice versa), then the total uncertainty in R has to be evaluated separately first in terms of δ_R and Δ_R.

That is, if the randomness in the R_i's are statistically independent, whereas the corresponding prediction errors are correlated, then the total randomness in R would be

$$\delta_R^2 \simeq \frac{1}{\mu_R^2} \sum_{i=1}^{n} c_i^2 \, (\delta_{R_i} \mu_{R_i})^2 \tag{26}$$

whereas,

$$\Delta_R^2 \simeq \Delta_{g_1}^2 + \frac{1}{\mu_R^2} \sum_{i=1}^{n} \sum_{j=1}^{n} c_i c_j (\Delta_{R_i} \mu_{R_i})(\Delta_{R_j} \mu_{R_j}) \tag{27}$$

in which:

Δ_{g_1} = c.o.v. representing the dispersive error in the model function $g_1(\ldots)$;

$c_i = \frac{\partial \hat{g}}{\partial R_i}$, evaluated at $\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n$.

From which the total uncertainty in R, therefore, is obtained as,

$$\Omega_R \simeq \sqrt{\delta_R^2 + \Delta_R^2}$$

The total uncertainty in S may be similarly obtained.

In order to relate the total probability concept described above to the notion of conditional survival or failure probability (described subsequently below), the implications of the above total probability may be explained also as follows.

In Fig. 11a is shown the probability density function of $\hat{R}$ and $\hat{S}$, representing only the randomness in R and S, with estimated mean-values $\bar{r}$ and $\bar{s}$, and respective c.o.v. δ_R and δ_S. Because of uncertainties in the estimated mean-values, $\bar{r}$ and $\bar{s}$, these may also be described as random variables $\bar{R}$ and $\bar{S}$ with corresponding PDF $f_{\bar{R}}(\bar{r})$ and $f_{\bar{S}}(\bar{s})$, respectively, as shown in Fig. 11b with respective c.o.v. Δ_R and Δ_S. Therefore, for specified values of $\bar{r}$ and $\bar{s}$, as shown in Fig. 11a, the failure or survival probability associated with $f_{\hat{R}}(r)$ and $f_{\hat{S}}(s)$ is conditional on the given values of $\bar{r}$ and $\bar{s}$; hence, a calculated probability is a conditional probability, or

$$p_F = P(R < S \mid \bar{R} = \bar{r}, \bar{S} = \bar{s}) \tag{28}$$

In light of the distributions for $\bar{R}$ and $\bar{S}$, as shown in Fig. 11b, the probability that the mean-value $\bar{S} = \bar{s}$ is only $f_{\bar{S}}(\bar{s})\, d\bar{s}$; similarly the probability of $\bar{R} = \bar{r}$ is $f_{\bar{R}}(\bar{r})\, d\bar{r}$.

In effect, we have two sets of probability distributions; one representing the randomness in R and S, whereas the other representing the errors in the predicted mean (or median) values. The probability of failure, or survival, may include all possible values of $\bar{R}$ and $\bar{S}$; the result is the total or expected probability,

$$p_F = \sum_{\text{all } \bar{r},\bar{s}} P(R < S \mid \bar{r},\bar{s}) \cdot P(\bar{R} = \bar{r}, \bar{S} = \bar{s})$$

$$\int_0^\infty \int_0^\infty \left[\int_0^\infty F_{\hat{R}|\bar{r}}(s) \cdot f_{\hat{S}|\bar{s}}(s)\, ds \right] f_{\bar{R}}(\bar{r}) \cdot f_{\bar{S}}(\bar{s})\, d\bar{r}\, d\bar{s} \tag{29}$$

Eq. 29 is actually equivalent to Eq. 1 or 4 with total uncertainties Ω_R and Ω_S. It may be observed that Eq. 29 is merely an application of the total probability concept described earlier in Sect. 2.4; this is appropriate when there is imperfect information as in the present case of the estimated mean-values $\bar{r}$ and $\bar{s}$.

3.2.2 The Conditional Probability Approach

Alternatively, the calculated probability of survival may represent only the effects of uncertainty associated with inherent randomness. However, such a probability depends on the parameters (specifically the mean or median values) of the underlying distribution. Because these parameters are subject to errors (i.e. prediction errors), the resulting probability of survival or failure also contains uncertainty, which may be represented by an interval or error bound on which the correct probability may lie. In this regard, it may be emphasized that the information normally available to assess the error bound on the calculated probability is often based largely or entirely on subjective judgments. For this reason, theoretical exactness in the determination of this error bound is seldom justified, for the reason that the available basis for its determination does not warrant exactness or accuracy, especially if such accuracy requires added complexity.

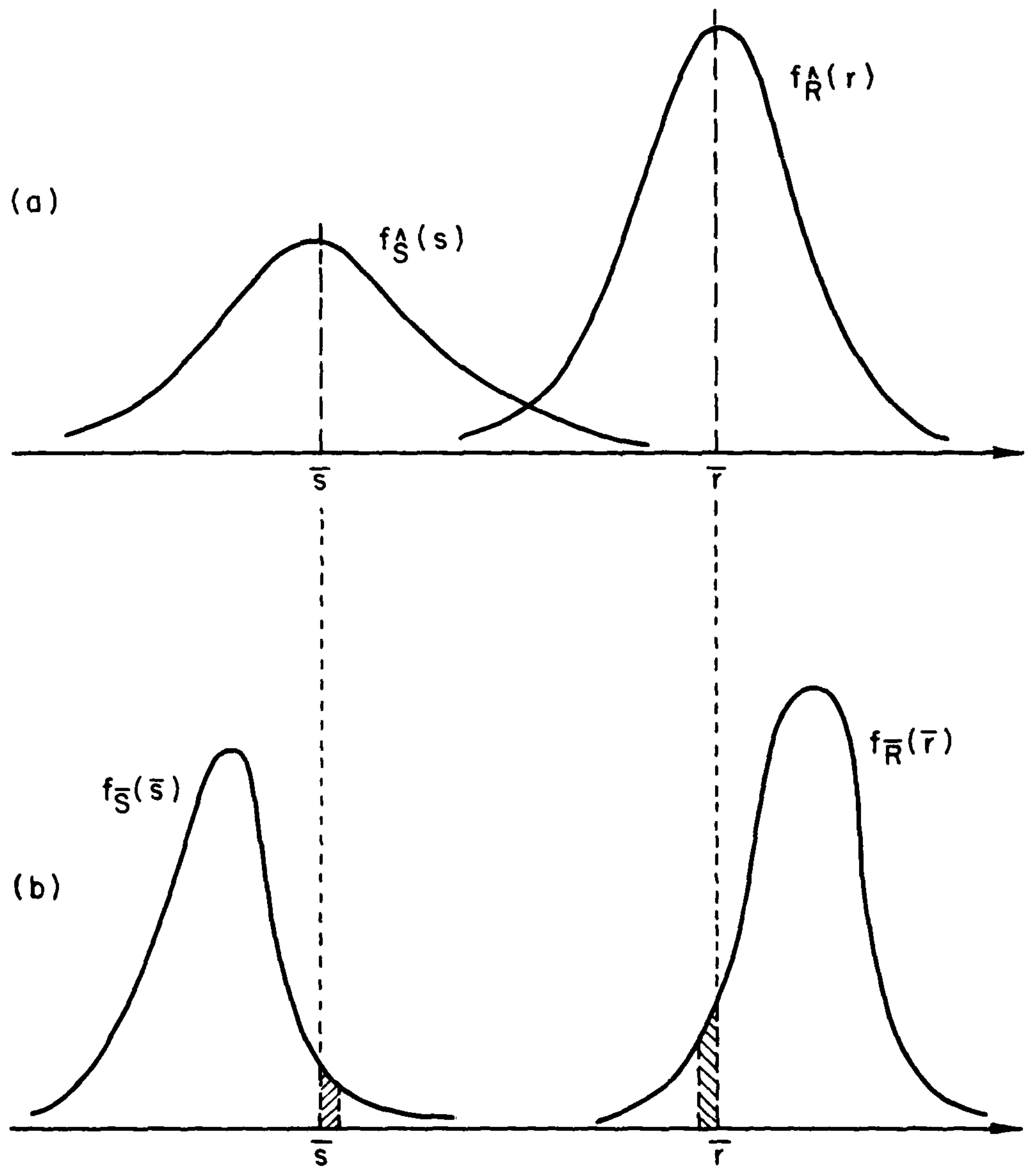


FIG. 11 (a) PDF OF $\hat{R}$ AND $\hat{S}$; (b) PDF OF $\bar{R}$ AND $\bar{S}$

Probability may be implemented also in a conditional sense; that is, because of uncertainty due to prediction errors, a calculated probability representing the effects of randomness only is conditional on the values of the parameters (e.g. $\bar{r}$ and $\bar{s}$) used in the calculation. In other words, p_S or p_F may represent only the consequence of uncertainty due to randomness, whereas in order to "cover" the uncertainty associated with prediction errors, conservative values of $\bar{r}$ and $\bar{s}$ may be specified. In this case, the resulting probability, therefore, is associated only with the probability density function (PDF) $f_{\hat{R}}(r)$ and $f_{\hat{S}}(s)$ conditional on conservative mean-values $\bar{r}_\ell$ and $\bar{s}_u$. The degree of conservatism may be designated by the cumulative probabilities associated with $\bar{s}_u$ and $\bar{r}_\ell$. In this approach, the probability of survival, therefore, is a <u>conditional probability</u> based on the probability distributions of $\hat{S}$ and $\hat{R}$ depicted in Fig. 12; i.e.

$$p_S - 1 - \int_0^\infty F_{\hat{R}|\bar{r}_\ell}(s) \cdot f_{\hat{S}|\bar{s}_u}(s)\, ds \qquad (30)$$

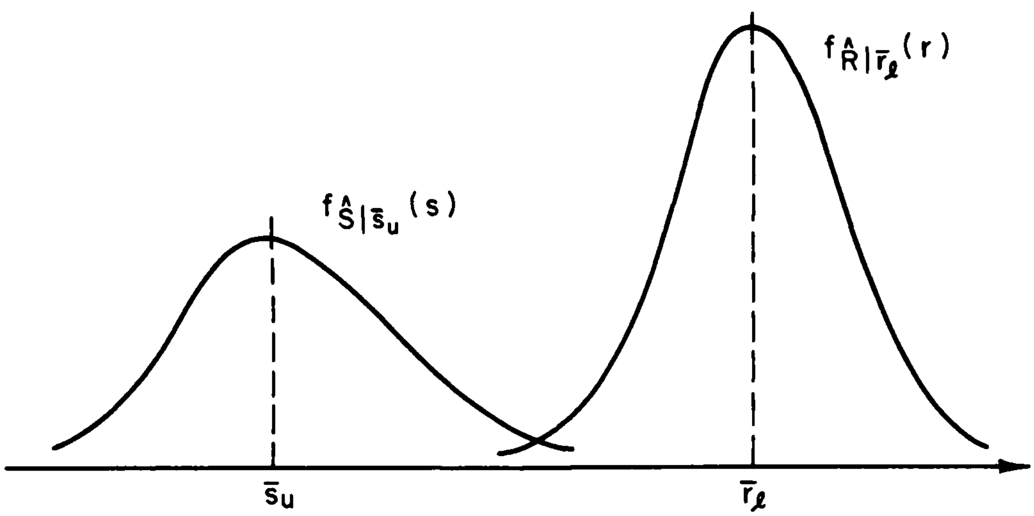


FIG. 12 PDF OF $\hat{R}$ AND $\hat{S}$ WITH SPECIFIED $\bar{r}_\ell$ AND $\bar{s}_u$

<u>Remarks</u> -- The conditional probability approach may be particularly appropriate when the mean or median values are largely unknown or contain very large uncertainty. In such cases, probabilities can be given conditionally on assumed values of the means or medians. However, if credible distributions for the mean or median values can be established, then the most realistic (rather than conservative) estimate of the true probability is the expected probability as given in Eq. 29.

3.2.3 Remarks on Deterministic Procedure

Much of engineering analysis and design, including applications in strategic problems, will remain largely deterministic. Probabilistic analysis, of course, is needed to properly assess and analyze the effects of uncertainties on the survival or failure of a given system. However, once such uncertainties have been assessed and their effects on survivability have been evaluated, the results may then be used in a purely deterministic format. After all, a structure or system may always be designed conservatively such that there is adequate resistance relative to a given weapon effect; it is in determining what constitutes adequate conservatism that probability concepts are important. Consequently, it is not surprising that the key role of probability is supplemental to existing deterministic methods, in the sense of providing a proper framework for the analysis and evaluation of uncertainties and of its effect on survivability.

Again, it may be emphasized (see Figs. 2 and 3), that the probability of failure (or survival) is a function of the central factor of safety. The central factor of safety, which is the ratio of $\overline{r}/\overline{s}$ or $\tilde{r}/\tilde{s}$ is a deterministic quantity; hence, once the degree of uncertainty and its effects have been evaluated, the main problem in the evaluation of survivability becomes a purely deterministic problem, or a problem that requires purely deterministic analysis for determining the mean (or median) values.

The premise of the present report, therefore, is that even though probabilistic concepts and methods are essential for engineering evaluation and design, the implementation of such concepts can be most effective if they are used to supplement or complement existing deterministic approaches to engineering. For this purpose, certain basic and generic relationships, however, must be developed. The approximate probabilistic analysis summarized herein should permit the development of the required relationships. Specifically, it is shown that for a given type of structure or system, whose level of uncertainty has been assessed, the probability of survival is a function of the central safety factor (ratio of medians or ratio of means). Conversely, design to achieve a desired probability of survival can be accomplished by using the appropriate central factor of safety. These relationships are subsequently developed below.

3.3 Generic Relationships

Once again, it may be emphasized that the probability of survival (or failure) depends on a few parameters characterizing the resistance and weapon effect. Specifically, it is a function of the relative positions of the resistance PDF, $f_{R(r)}$, and the PDF of the weapon effect, $f_S(s)$, and of the respective uncertainty measures Ω_R and Ω_S.

In Eq. 4 for lognormal distribution, or Eq. 5 for normal distribution, it is shown explicitly that the survival probability p_S is a function of the central safety factor, $\tilde{\theta}$ or $\overline{\theta}$, and the degree of uncertainty, ζ or Ω; hence, in evaluating the probability of survival, or failure, the main task involves the determination of $\tilde{\theta}$ or $\overline{\theta}$, once ζ or Ω has been evaluated. The determination of $\tilde{\theta}$ or $\overline{\theta}$, of course, requires only deterministic analysis since it involves the calculation of the median or mean values.

The determination of the probability of kill is similar to that of the probability of survival. As in the case of the probability of survival which is a function of the central safety factor, the probability of kill is also a function of a similar factor which may be called "the overkill factor", that represents the relative positions between the PDF of the weapon effect and the PDF of the resistance. Therefore, just as the survival probability involves the evaluation of the central safety factor, the probability of kill requires the evaluation of the central overkill factor.

Design Relationships -- Design, in the context used herein, means the determination of the required resistance in order to achieve a specified level of survivability. Specifically, this involves the determination of the required resistance in order to achieve an acceptable or specified survival probability p_S. Again, this design can be determined through the application of an appropriate central factor of safety as follows:

Required median resistance, to resist an applied median weapon effect $\tilde{s}$, is

$$\tilde{r} = \tilde{\theta}\,\tilde{s}$$

in which the appropriate safety factor is such that it corresponds to a specified probability of survival p_S. This probability-based safety factor can be obtained from inversion of Eq. 4, obtaining

$$\tilde{\theta} = e^{\beta\zeta}$$

where;

$\beta = \Phi^{-1}(p_S)$, the value of the standard normal variate at cumulative probability p_S, and

$$\zeta = \sqrt{\zeta_R^2 + \zeta_S^2}$$

Similarly, in designing a weapon system to achieve a desired probability of kill p_K, the required median weapon effect can be obtained by using the appropriate median overkill factor $\tilde{\theta}_K$, obtaining

$$\tilde{s} = \tilde{\theta}_K\,\tilde{r}$$

where, $\tilde{r}$ is the estimated median resistance of the enemy target. The required median overkill factor is determined from

$$\tilde{\theta}_K = e^{\beta\zeta}$$

where, in this case,

$$\beta = \Phi^{-1}(p_K)$$

The relationships referred to above are developed more explicitly below for the case involving lognormal distributions.

3.3.1 Evaluation of Expected Probability of Survival or Kill*

The basic relationships and procedures needed for survivability evaluation as well as survivability design are summarized below for lognormal distributions. The lognormal distributions are used here for purposes of illustration; in an actual problem, these distributions may or may not be the most appropriate. If not, relationships that are similar to those illustrated here for the lognormal distributions may also be developed. The main steps in the evaluation of survival or kill probability may be outlined as follows:

Step 1 -- Determine median resistance $\tilde{r}$, and median weapon effect $\tilde{s}$; and evaluate the available safety factor.

$$\tilde{\theta}_S = \tilde{r}/\tilde{s}$$

or overkill factor,

$$\tilde{\theta}_K = \tilde{s}/\tilde{r}.$$

Step 2 -- Evaluate uncertainties in R and S in terms of c.o.v. Ω_R and Ω_S, and obtain

$$\Omega \approx \sqrt{\Omega_R^2 + \Omega_S^2}$$

and,

$$\zeta = \sqrt{\ell n(1+\Omega^2)}$$

Step 3 -- Calculate probability of survival,

$$p_S = \Phi\left(\frac{\ell n\ \tilde{\theta}_S}{\zeta}\right) \qquad (4)$$

or probability of kill,

$$p_K = \Phi\left(\frac{\ell n\ \tilde{\theta}_k}{\zeta}\right) \qquad (4a)$$

The relationship between the survival (or kill) probability and the required safety (or overkill) factor may be portrayed graphically on the lognormal probability paper as shown earlier in Fig. 4 for various values of ζ.

In other words, to evaluate the survival probability for a given structure or facility under a stipulated enemy attack, we evaluate the environment expected from the enemy weapon (i.e. $\tilde{s}$ and Ω_S); also, for a given structure or facility we determine the

*Any reference here to targeting is presented only to show the similarity between the problems of target analysis and survivability evaluation.

median resistance $\tilde{r}$ and associated uncertainty Ω_R. From these the central safety factor $\tilde{\theta}_S$ is obtained, and the survival probability p_S is calculated through Eq. 4 or read from Fig. 4 with the appropriate value of ζ.

Similarly, in the case of evaluating the kill probability, we estimate the resistance $\tilde{r}$ and Ω_R of the enemy installation, utilizing intelligence information when it is available; also, we evaluate the median weapon effect $\tilde{s}$ and associated uncertainty Ω_S from the weapon system to be employed for the attack. On this basis, the median overkill factor is determined as $\tilde{\theta}_k = \tilde{s}/\tilde{r}$. The probability of kill is determined either with Eq. 4a or from Fig. 4 corresponding to the given ζ.

3.3.2 Steps in Design to Achieve Specified Survivability or Kill Potential

In the case of design to insure survivability, the objective is to design a defense system in order to achieve a desired probability of survival p_S against a stipulated enemy weapon threat, whereas from an offensive standpoint it is to plan a targeting strategy in order to achieve a desired probability of kill p_K. (Any reference here to targeting is simply to show the similarity in the problem with that of survivability design).

In these cases, the desired p_S or p_K would be specified, and the uncertainty associated with R and S would be evaluated. Eq. 4 is then used to determine the required median safety factor $\tilde{\theta}_S$, or median overkill factor $\tilde{\theta}_K$, as the case may be. Purely deterministic analysis is then used to determine the median resistance $\tilde{r} = \tilde{\theta}_S\tilde{s}$ required to insure defense survival with probability p_S, or the required weapon system to deliver $\tilde{s} = \tilde{\theta}_K \tilde{r}$ in order to insure kill with probability p_K.

The specific steps in the design process, therefore, may be outlined as follows:

Step 1 -- Specify or prescribe the desired survival probability p_S, or kill probability p_K, as the case may be.

Step 2 -- Determine the median safety factor $\tilde{\theta}_S$, or median overkill factor $\tilde{\theta}_K$, necessary to achieve p_S or p_K; i.e.

$$\tilde{\theta}_S = e^{\beta\zeta}$$

where, $\beta = \Phi^{-1}(p_S)$

Similarly,

$$\tilde{\theta}_K = e^{\beta\zeta}$$

where,

$$\beta = \Phi^{-1}(p_K)$$

Step 3 -- The required median resistance $\tilde{r}$ is

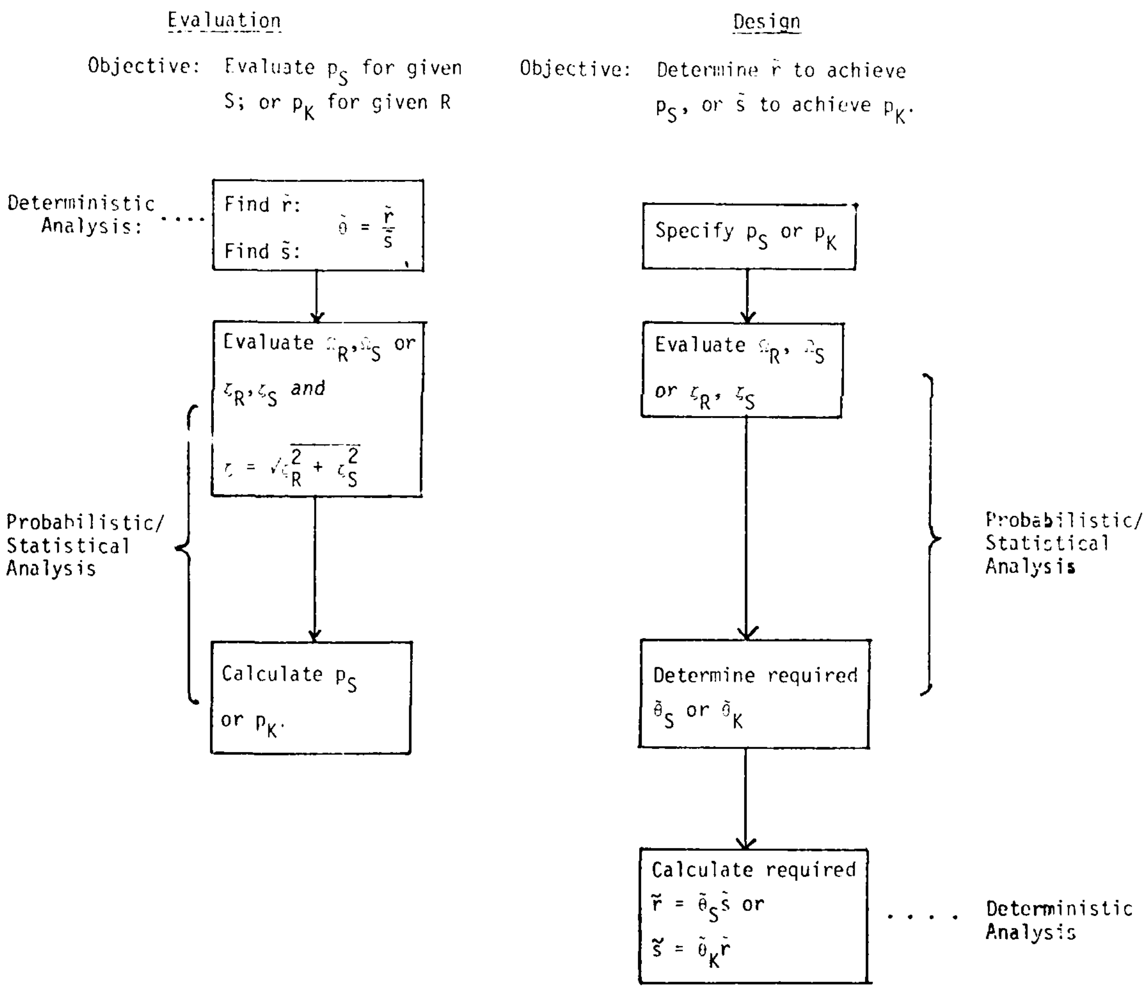
$$\tilde{r} = \tilde{\theta}_S\tilde{s} ;$$

whereas, the required median weapon effect is,

$$\tilde{s} = \tilde{\theta}_K \tilde{r} \ .$$

The processes of survivability (or kill) evaluation and survivability design (or target planning) are summarized in Table 2. Also, in Table 2, the places where statistical analyses are required are indicated; the remaining steps, therefore, involve purely deterministic analysis.

TABLE 2: Outline of Survivability Evaluation and Design



3.4 Survivability and Design of Underground Tunnels

The survivability and design of underground tunnels to ground-transmitted pressures from a nuclear burst will illustrate the implementation of the basic probability principles in the evaluation and design of a nontrivial strategic structural problem. This application should also serve to illustrate and emphasize, in particular, how probability methods may be used in conjunction with necessary deterministic analysis. The same problem was recently discussed in a report by Boeing (1977); therefore, liberal references to this report will be made for some of the relationships used herein pertaining to underground tunnels.

The survivability or vulnerability of underground tunnels to nuclear weapon effect is generally considered in terms of its resistance to uniform hydrostatic pressures. Such pressures are invariably determined or derived through available information for particle velocities at the same depth; pertinent equations for particle velocities, developed empirically from nuclear test data, were discussed and described in an earlier example in Sect. 2.5.2. The compressive pressure P is then determined by

$$P = \rho c u \tag{31}$$

where: u = particle velocity,

c = wave propagation speed,

ρ = mass density of the material.

The free-field particle velocity may be given by an equation of the following form:

$$u = C(KW)^a R^b, \text{ (in m/sec)} \tag{32}$$

in which;

R = the slant range, in meters;

W = weapon yield, in kilotons;

K = energy coupling factor;

C = coefficient determined from linear regression;

a, b = exponents determined from linear regression.

The resistance P_o of a lined or unlined tunnel to free-field pressure loads may be estimated with the Newmark equation as follows (Boeing, 1977):

$$P_o = \frac{K_{sr}+1}{2(K_{sr}-1)} \left[S_{ri}^{\left(\frac{K_{sr}-1}{K_{sr}+1}\right)} \cdot Y_{ri}^{\left(\frac{2}{K_{sr}+1}\right)} - \left(\frac{u_r}{K_{sr}-1}\right) \right] \tag{33}$$

where:

$$S_{ri} = \frac{\varepsilon_r E_r}{(1-\nu_r^2)} - \frac{\sigma_{rc}(1-2\nu_c)}{(1-\nu_c)} \text{ ; if failure occurs in the rock;}$$

$$S_{ri} = \frac{E_r(1-\nu_c^2)}{E_c(1-\nu_r^2)} \left[\frac{r_a^2}{r_{co}^2} \left\{ \frac{\varepsilon_c E_c}{(1-\nu_c^2)} - \frac{\sigma_{ra}(1-2\nu_c)}{(1-\nu_c)} \right\} \right.$$

$$\left. + \frac{\sigma_{rc}(1-2\nu_c)}{(1-\nu_c)} \right] - \frac{\sigma_{rc}(1-2\nu_r)}{(1-\nu_r)} \text{ ; if failure occurs in the liner,}$$

and, $\quad Y_{ri} = \sigma_{ur} + (K_{sr}-1)\,\sigma_{rc};$

in which:

σ_{ra} = radial stress at the concrete/steel interface;

$$= \frac{h_s \cdot f_y}{r_a}$$

σ_{rc} = radial stress at the rock/concrete interface;

$$= \frac{1}{(K_{sc}-1)} \left[\{f_c' + (K_{sc}-1)\,\sigma_{ra}\} \left(\frac{r_{co}}{r_a}\right)^{(K_{sc}-1)} - f_c' \right]$$

σ_{ur} = unconfined compressive strength of rock (psi);

f_y = yield stress of steel liner (psi);

f_c' = unconfined compressive strength of concrete (psi);

ε_c = circumferential strain at concrete/steel interface for liner failure;

ε_r = circumferential strain at rock/concrete interface for rock failure;

E_r = equivalent elastic modulus of rock (psi);

E_c = equivalent elastic modulus of concrete (psi);

K_{sr} = friction-dependent constant for rock;

K_{sc} = friction-dependent constant for concrete;

ν_r = Poisson's ratio for rock;

ν_c = Poisson's ratio for concrete;

r_{co} = radius to rock/concrete interface (in);

r_a = radius to concrete/steel interface (in);

h_s = thickness of steel liner (in).

Derministic Analysis -- The deterministic part of the evaluation involves the determination of the median safety factor as follows:

For the purpose of numerical illustration, consider an underground tunnel as shown in Fig. 13 with an inside radius of 84", lined with a 12" concrete and 1.25" steel liner (see Boeing, 1977).

The specific material parameters and tunnel dimensions are as follows (Boeing, 1977):

Parameter	Mean Value	c.o.v.
σ_{ur}	4,000 psi	0.48
f_y	52,400 psi	0.06
f'_c	7,030 psi	0.056
ε_c	0.04	0.81
E_c	5.5×10^6 psi	0.036
E_r	6.53×10^5 psi	0.26
K_{sc}	4.1	0.34
K_{sr}	1.22	0.17
ν_c	0.30	0
ν_r	0.39	0
r_a	84 in.	0
r_{co}	96 in.	0
h_s	1.25 in.	0

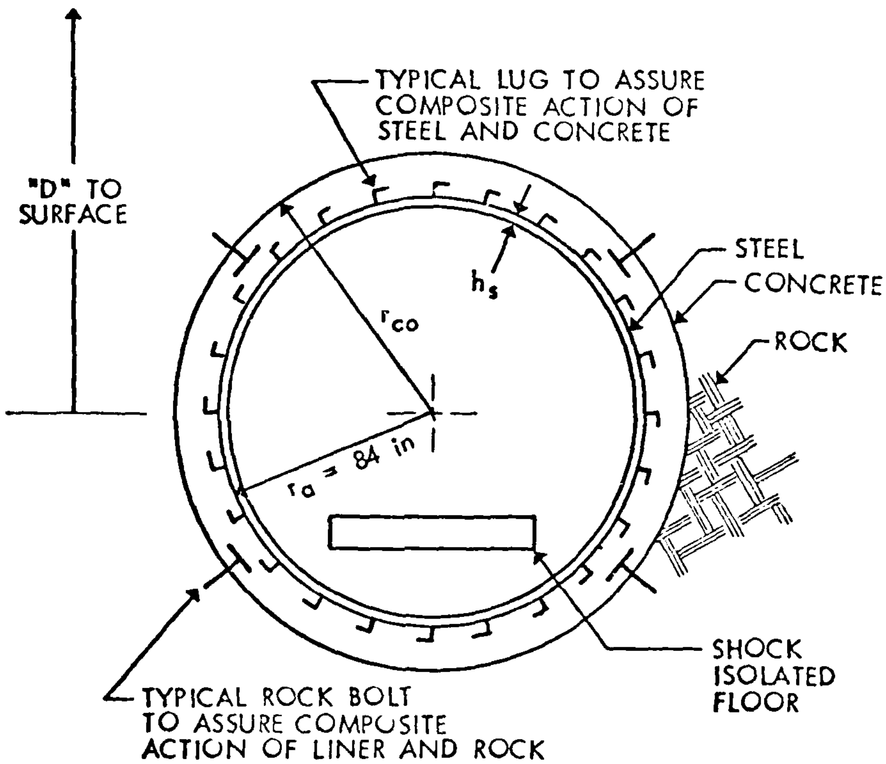


Fig. 13 Deep-buried Tunnel Configurations--
After Boeing (1977)

With the above parameters the median resisting pressures of the tunnel, obtained with the Newmark equation, can be summarized as follows:

Median Resisting Pressure P_o of Tunnel

Failure Mode	Material: Tuff	Material: Hard Rock
Liner	7,790 psi	79,300 psi
Rock	10,450 psi	101,100 psi

These resisting pressures are indpendent of the depth of burial D.

Based on the particle velocity of Perret and Bass (1975) for wet tuff, the equation for the median applied pressure is given by Boeing (1977):

$$\tilde{P} = 5.6 \times 10^6 \, (KW)^{0.52} \, R^{-1.56}$$

whereas in hard rock the corresponding equation is (Boeing 1977):

$$\tilde{P} = 34.1 \times 10^6 \, (KW)^{0.5733} \, R^{-1.72}$$

The slant range R is,

$$R = \sqrt{D^2 + L^2}$$

in which L is the miss distance, whose median is the CEP; i.e. $\tilde{L}$ = CEP. Denoting $\rho = \frac{CEP}{D}$, the median range for a given depth D is

$$\tilde{R} = \sqrt{D^2 + \tilde{L}^2} = D \sqrt{1 + \rho^2}$$

Consider a surface burst with a yield of W = 5 megatons = 5,000 kilotons, and a weapon accuracy of CEP = 500 meters.

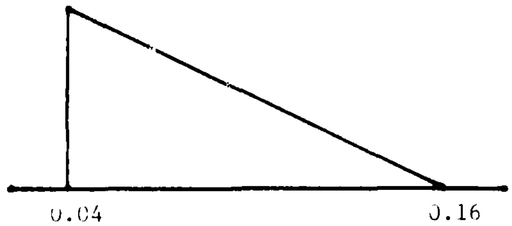
If the tunnel is at a depth of 1000 meter in wet tuff

$$\rho = 0.5$$

and,

$$\tilde{R} = 1000 \sqrt{1 + 0.5^2} = 1118 \text{ m}$$

According to Cooper (1973), the coupling factor K for surface contact burst is 0.04, whereas for shallow buried burst K = 0.16. For a nominally surface burst, the appropriate value of K may be assumed to range between these two limits, although it is more likely to be closer to 0.04. On the basis of these assumptions, a left triangular distribution may be assumed for K, as shown below.



The mean-value of K, therefore, is

$$\bar{K} = 0.04 + \frac{1}{3}(.12) = 0.08$$

Hence, the median pressure is,

$$\tilde{P} = 5.6 \times 10^{6} (.08 \times 5{,}000)^{0.52} (1118)^{-1.56}$$

$$= 2217 \text{ psi}$$

At the depth of 1000 m, the median safety factor for the tunnel, therefore, is

$$\tilde{\theta} = \frac{7790}{2217} = 3.51$$

Thus far, the analysis is entirely deterministic.

Probabilistic/Statistical Analysis -- The probabilistic/statistical part of the evaluation involves the determination of the pertinent uncertainty measures, which are as follows:

Using the partial derivatives of the Newmark equation derived by Boeing (1977), the contributions of the various variables to the total uncertainty in P_o were obtained for wet tuff and hard rock as follows:

Contributions to $\Omega^2_{P_o}$ (Liner Failure Mode)

Variable	Material: Wet Tuff	Material: Hard Rock
σ_{ur}	0.1570	0.0914
σ_{ra}	0	0
σ_{rc}	0.0130	0.0491
ε_c	0.3013	0.2790
K_{sr}	0.1186	0.0633
E_r	0.0903	0.0901
Total =	0.342	0.349

Contributions to $\Omega_{P_o}^2$ (Rock Failure Mode)

Variable	Material: Wet Tuff	Material: Hard Rock
σ_{ur}	0.0965	0.2145
σ_{rc}	0	0
ε_r	0.2706	0.1715
E_r	0.0866	0.0719
K_{sr}	0.1488	0.1109
Total =	0.305	0.335

On the basis of the above results, the total uncertainty in the estimated resisting pressure P_o of the tunnel in wet tuff is $\Omega_{P_o} = \sqrt{0.342} = 0.58$.

The Newmark equation was developed in terms of the resistance to free-field pressure and, therefore, is based on the assumption of purely hydrostatic pressure loading. This may be reasonable if the advancing shock wave is rapid enough to engulf the tunnel before there is any appreciable distortion of the tunnel. In such cases, it may be reasonable to assume that there is no bias in the Newmark equation and that the total dispersive uncertainty is $\Omega_{P_o} = 0.58$. Then,

$$\zeta_{P_o} = \sqrt{\ln(1 + \Omega_{P_o}^2)} = \sqrt{\ln(1 + .58^2)}$$

$$= 0.54$$

In the case of the applied pressure, according to Perret and Bass (1975), the variabilities in the exponents a and b of Eq. 32 are small compared to those in the coefficient C and coupling factor K; thus, the uncertainties in a and b may be neglected. On this basis, the total uncertainty in P is,

$$\Omega_P \approx \sqrt{\Omega_C^2 + (a\ \Omega_K)^2 + (b\ \Omega_R)^2} \tag{34}$$

In the following, it will be assumed that there is no model bias or additional dispersive uncertainty in the pressure equation, Eq. 32, as developed by Perret and Bass (1975). Also, using the variance factors evaluated by Perret and Bass (1975) as described earlier in Sect. 2.5.2, which gave a variance factor of 1.56 for wet tuff, we obtain

$$\zeta_C = \ln 1.56 = 0.445$$

and,

$$\Omega_C = \sqrt{e^{(.445)^2} - 1} = 0.47$$

In an actual burst, it is difficult to anticipate the burst condition and, therefore, significant uncertainty in the coupling factor K can be expected. As indicated earlier, the burst condition may be assumed to range betwen the surface contact and shallow buried conditions; on this basis, and the left triangular distribution assumed above, the c.o.v. associated with K may be evaluated as

$$\Omega_K = 0.707\left(\frac{0.16 - 0.04}{0.16 + 2 \times 0.04}\right) = 0.35$$

The slant range R is a function of the miss distance L, and the depth of burial D of the tunnel. In evaluating the survivability or vulnerability of a defensive system, the depth D is, of course, known; therefore, there is no uncertainty. The miss distance L of an enemy weapon burst, however, will have variability. This will depend on the accuracy of the weapon, specified in terms of its CEP. On this basis, the c.o.v. of the slant range R is evaluated as follows:

Again, denoting $\rho = \frac{CEP}{D}$, and recognizing that in the case of the miss distance, its median $\tilde{L}$ = CEP, the median range becomes

$$\tilde{R} = D\sqrt{1 + \rho^2}\,.$$

Also, the variance of R is, by first-order approximation

$$\sigma^2 \approx \frac{\overline{L}^2\,\sigma_L^2 + \overline{D}^2\,\sigma_D^2}{(\overline{L}^2 + \overline{D}^2)}$$

Replacing $\tilde{L}$ for L (i.e. using the median $\tilde{L}$ in place of the mean $\overline{L}$ in the above equation), the variance of R becomes

$$\sigma_R^2 \quad \frac{1}{1+\rho^2}\,(\sigma_D^2 + \rho^2\,\sigma_L^2)$$

But, $\sigma_D = 0$; hence, the standard deviation of R is

$$\sigma_R = \frac{\rho}{1+\rho^2}\,\sigma_L$$

from which, the c.o.v. of R is

$$\Omega_R \quad \frac{\sigma_R}{\tilde{R}} = \frac{\rho}{\sqrt{1+\rho^2}} \cdot \frac{\sigma_L}{D\sqrt{1+\rho^2}}$$

but, σ_L = 0.849 CEP; hence,

$$\Omega_R = \frac{0.849\ CEP}{D}\left(\frac{\rho}{1+\rho^2}\right) = 0.849\left(\frac{\rho^2}{1+\rho^2}\right)$$

At the depth of D = 1,000 meters, and with the indicated CEP = 500 meters, ρ = 0.5; hence $\Omega_R = 0.849\ (\frac{.5^2}{1+.5^2}) = 0.17$

Therefore, using a = 0.52 and b = 1.56 in Eq. 34, the total c.o.v. in the calculated applied pressure at depth D = 1,000 meters is

$$\Omega_p = \sqrt{0.47^2 + (0.52\text{x}0.35)^2 + (1.56\text{x}0.17)^2} = 0.57$$

and,

$$\zeta_p = \sqrt{\ln(1 + 0.57^2)} = 0.53$$

Hence,

$$\zeta = \sqrt{\zeta_{p_o}^2 + \zeta_p^2} = \sqrt{0.54^2 + 0.53^2} = 0.76$$

and the survival probability of the tunnel in wet tuff, therefore, is

$$p_S = \Phi(\frac{\ln 3.51}{0.76}) = \Phi(1.65)$$

$$= 0.950$$

or p_F = 0.050.

Following the procedure illustrated above, the survival probabilities for other depths of the same tunnel, subjected to the same weapon yield of W = 5MT and CEP = 500 m, were also evaluated; the results of these calculations are summarized below.

Summary of Survivability Calculations (w = 5 MT)

		Deterministic Analysis			Probabilistic/Statistical Analyses					
Depth, m	$\rho = \frac{CEP}{D}$	P(psi)	$\bar{P}_o$(psi)	$\dot{\theta}$	Ω_R	Ω_P	ζ_{P_o}	ζ_P	ζ	p_S
100	5.0	7544	7790	1.03	0.82	1.37	0.58	1.03	1.16	0.512
200	2.5	6928	"	1.12	0.73	1.25	"	0.97	1.11	0.540
500	1.0	4530	"	1.72	0.42	0.83	"	0.72	0.90	0.726
1000	0.5	2217	"	3.51	0.17	0.57	"	0.53	0.76	0.950
1500	0.333	1291	"	6.03	0.08	0.52	"	0.49	0.73	0.993
2000	0.25	853	"	9.13	0.05	0.51	"	0.48	0.72	0.999
3000	0.167	465	"	16.75	0.02	0.50	"	0.47	0.72	> 0.999

In the above table, the calculations involving purely deterministic analyses are distinguished from those that require probabilistic/statistical analyses. Observe also that for a given CEP, the probabilistic/statistical part of the evaluations will remain unchanged; only the deterministic part of the evaluations will be altered. For example, to obtain similar results for other weapon yields of the same tunnel will require the calculation of the median pressures $\tilde{P}$, and evaluation of the corresponding safety factors $\tilde{\vartheta}$. The results for other weapon yields may be summarized as follows:

Depth, m	p	Pressure, $\tilde{P}$, for Yield W					$\tilde{\vartheta}$ for Yield W				
		10MT	15MT	20MT	25MT	50MT	10MT	15MT	20MT	25MT	50MT
100	5.0	[illegible]	13,356	15,511	17,420	24,979	0.72	0.58	0.50	0.45	0.31
200	2.5	9,934	12,266	14,245	16,000	22,940	0.78	0.64	0.55	0.49	0.34
500	1.0	6,495	[illegible]	9,314	10,460	14,999	1.20	0.97	0.84	0.74	0.52
1000	0.5	3,178	3,924	4,558	5,110	7,340	2.45	1.99	1.71	1.52	1.06
1500	0.333	1,851	2,286	2,655	2,981	4,275	4.21	3.41	2.95	2.61	1.82
2000	0.25	1,224	1,511	1,755	1,971	2,826	6.36	5.16	4.44	3.95	2.76
3000	0.167	667	824	956	1,074	1,540	11.68	9.45	8.15	7.25	5.06

The resulting survival probabilities, therefore, would be as follows:

Depth, m	p_S for Yield W					
	5MT	10MT	15MT	20MT	25MT	50MT
100	0.512	0.390	0.320	0.274	0.245	0.156
200	0.540	0.413	0.345	0.295	0.261	0.166
500	0.726	0.579	0.488	0.425	0.371	0.233
1000	0.950	0.881	0.818	0.761	0.709	0.532
1500	0.993	0.976	0.954	0.929	0.905	0.794
2000	0.999	0.995	0.989	0.981	0.972	0.921
3000	> 0.9999	0.9997	0.999	0.998	0.997	0.988

The above results can be portrayed graphically (see Fig. 14) to show the relation between the survival probability and depth of burial D of this particular tunnel in wet tuff.

On the basis of the above results, the depth D required to insure a given survival probability p_S against various weapon yield W may be developed. For example, from Fig. 14, in order to insure a 90% survivability, the required depths D of the tunnel for given W are approximately as follows:

W, MT CEP = 500m	Required Depth, D	
	p_S = 90%	p_S = 50%
5	820 m	80 m
10	1050 m	360 m
15	1200 m	520 m
20	1350 m	610 m
25	1480 m	690 m
50	1860 m	960 m

The required required depths to achieve 50% survival probability are also given above. These relations are also shown graphically in Fig. 15.

Survivability Design -- If the appropriate curves, such as Figs. 14 and 15 are available, the required depth of burial may be obtained directly from such curves to insure a specified probability of survival. The required tunnel depth, of course, may also be determined directly to satisfy a prescribed survival probability; we illustrate this as an example of the problem of design for survivability.

For the same tunnel in wet tuff, suppose that the threat is a penetrating weapon of W = 10MT with CEP = 200 m. A shallow-buried burst condition is appropriate; assume a mean coupling factor $\overline{K}$ = 0.16, and a c.o.v. Ω_K = 0.35.

In this case, since D is unknown, ρ is also unknown. So a trial-and-error procedure is necessary.

Suppose the prescribed survival probability is p_S = 0.90. The trial-and-error procedure may proceed as follows:

Assume a trial depth D = 1000 m.

Thus, $\rho = 0.20$

Then,

$$\Omega_R = 0.849\left(\frac{0.2^2}{1+0.2^2}\right) = 0.03$$

and,

$$\Omega_P = \sqrt{0.47^2 + (0.52 \times 0.35)^2 + (1.56 \times 0.03)^2}$$

$$= 0.51$$

From which, $\zeta_P = 0.48.$

The uncertainty in P_o remains Ω_{P_o} = 0.58 and ζ_{P_o} = 0.54. Hence,

$$\zeta = \sqrt{0.48^2 + 0.54^2} = 0.72$$

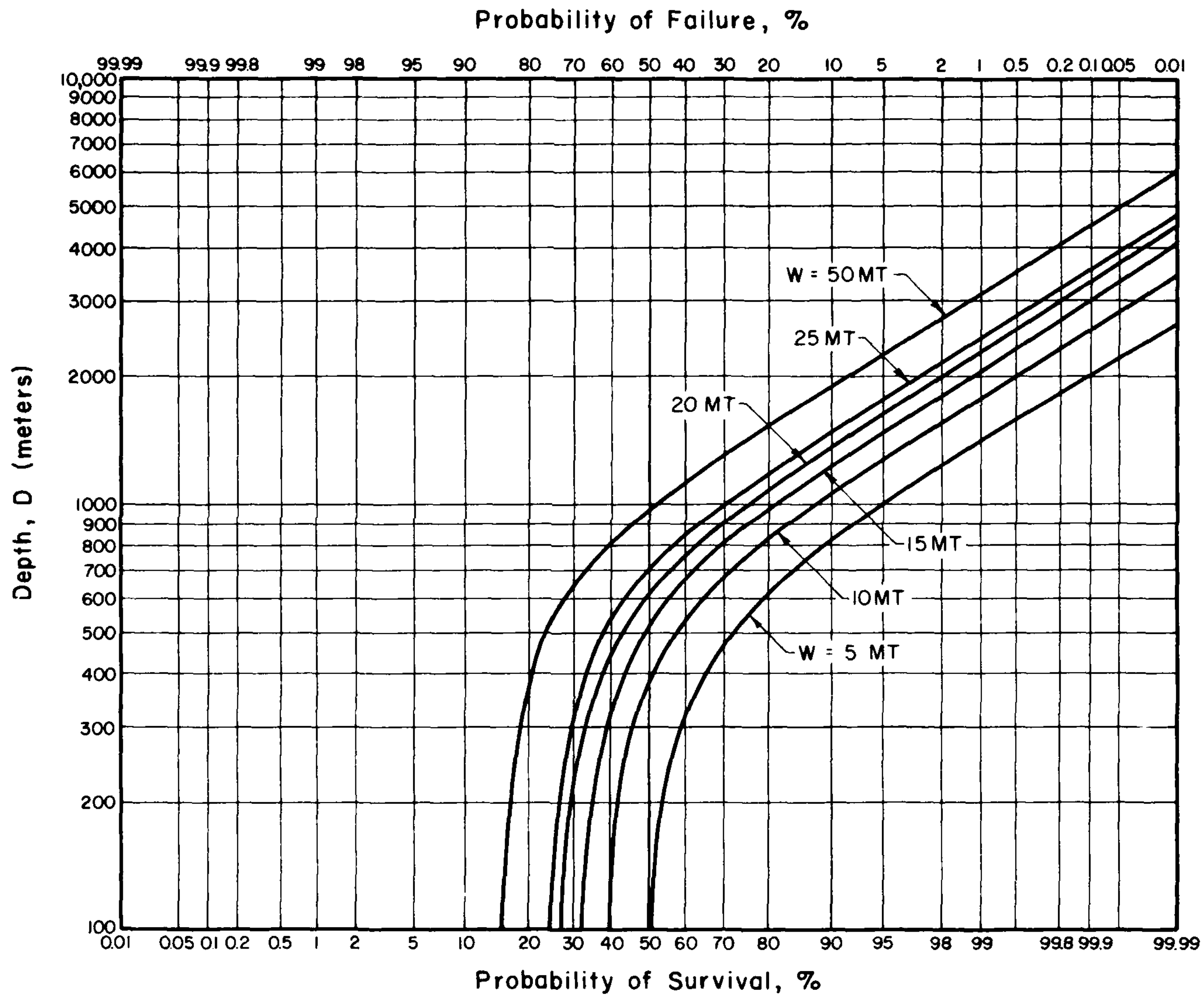


FIG. 14 DEPTH OF BURIAL VS SURVIVAL PROBABILITY (CEP = 500m)

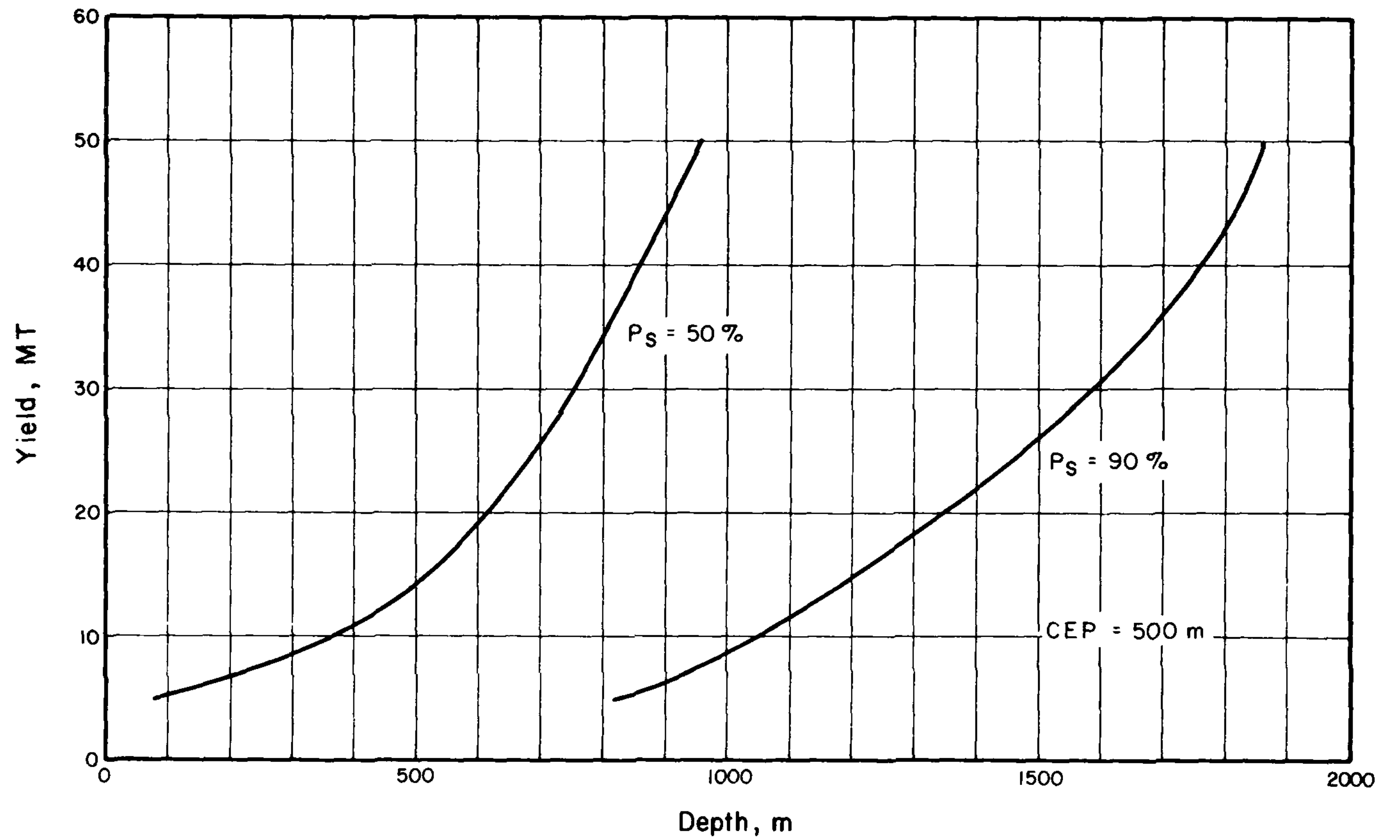


FIG. 15 DEPTH VS WEAPON YIELD FOR GIVEN P_S

For a 90% survival probability, the required median safety factor, Eq. 6 is

$$\tilde{\theta} = e^{1.28 \times 0.72} = 2.51$$

Therefore,

$$\frac{7790}{5.6 \times 10^6 (KW)^{0.52} (D\sqrt{1+\rho^2})^{-1.56}} = 2.51$$

From which the required depth is,

$$D = \left[\frac{2.51 \times 5.6 \times 10^6 (KW)^{0.52}}{7790\ (1+\rho^2)^{0.78}} \right]^{\frac{1}{1.56}}$$

$$= 1400 \text{ m.}$$

In the next trial, assume D = 1400 m.

$$\rho = \frac{200}{1400} = 0.14$$

$$\zeta_R = 0.849 \left(\frac{.14^2}{1+14^2} \right) = 0.02$$

$$\Omega_P = \sqrt{.47^2 + (.52 \times .35)^2 + (1.56 \times .02^2}$$

$$= 0.50;$$

$$\zeta_P = 0.47$$

$$\zeta \quad \sqrt{.47^2 + .54^2} = 0.72$$

Therefore,

$$\tilde{\theta} = e^{1.28 \times 0.72} = 2.51 \quad \text{(the same as before)}$$

Hence, the required depth to achieve $p_S = 0.90$ is

$$D = \underline{1400 \text{ m}}.$$

Target Planning -- For offensive purposes, in order to insure the destruction of an enemy installation (e.g. a tunnel) with a probability of kill, p_K, the necessary weapon system may be determined as follows:

Suppose that the target is a tunnel (of similar configuration as the one discussed earlier) at a depth D = 500 meters; and that the material is believed to be wet tuff. The median resisting pressure, therefore, is $\tilde{P}_o$ = 7790 psi.

If shallow penetrating weapons are to be used, with CEP of 200 m, the weapon yield W needed to achieve a kill probability of p_K = 0.75 is determined as follows:

With D = 500 m, and CEP = 200 m,

$$\rho = 0.4$$

Then,

$$\tilde{R} = 500\sqrt{1 + (0.4)^2} = 539 \text{ m.}$$

and,

$$\Omega_R = 0.849 \left(\frac{0.4^2}{1 + 0.4^2}\right) = 0.12$$

Hence,

$$\Omega_P = \sqrt{0.47^2 + (0.52 \times 0.35)^2 + (1.56 \times 0.12)^2}$$

$$= 0.54$$

and,

$$\zeta_P = 0.51$$

Again, with $\Omega_{P_o} = 0.58$, and $\zeta_{P_o} = 0.54$,

$$\zeta = \sqrt{0.51^2 + 0.54^2} = 0.74$$

Therefore, in order to achieve p_K = 0.75,

$$\frac{\ln \tilde{\theta}_K}{0.74} = 0.68$$

From which we obtain the required overkill factor;

$$\tilde{\theta}_K = e^{0.68 \times 0.74} = 1.65$$

Hence, the median pressure necessary to destroy the tunnel is,

$$\tilde{P} = 1.65\ \tilde{P}_o = 1.65x7790 = 12,850\ psi$$

From Eq. 32 (with the appropriate coefficients for wet tuff), the required weapon yield is,

$$W = \frac{1}{0.16}\left(\frac{\tilde{P}\ \tilde{R}^{1.56}}{5.6x10^6}\right)^{1.92}$$

$$= 6.9x10^{-13}\ \tilde{P}^{1.92}\ \tilde{R}^{3.00}$$

$$= 6.91x10^{-13}\ (12,850)^{1.92}\ (539)^{3.00}$$

$$= 8,400\ KT$$

$$= \underline{8.4\ MT}.$$

3.5 Survivability of Equipment to Ground Shocks

Mechanical and electrical equipments that are housed within a blast-resistant building or underground shelter are primarily vulnerable to ground shocks. Depending on the natural frequency of the equipment and its mounting, relative to those of the ground motions, a piece of equipment may be most vulnerable to the peak acceleration, velocity, or displacement of the forcing motions.

The survivability or vulnerability of equipment, obviously, will also depend on the level of motions that a particular piece of equipment can tolerate before it becomes inoperative or damaged (i.e. the fragility limit). Again, this will depend on the characteristics of the equipment and its susceptibility to acceleration, velocity, or displacement; however, information on the fragility limits of equipments (if available) are invariably in terms of its tolerance to peak acceleration. Accordingly, in evaluating the survivability of equipments, the evaluation may be performed on the basis of acceleration tolerances only.

As expected, there is considerable uncertainty in such survivability/vulnerability evaluations. Aside from uncertainty underlying the prediction of the ground accelerations, there is also major sources of uncertainty associated with the response of an equipment to a specified free-field ground motion environment. Moreover, if the equipment is mounted or attached to the interior of a building or structure, the motion to which the equipment is subjected has been filtered through the structure and, therefore, could be quite different from that of the free-field. The analysis or prediction of such in-structure motions may contain further uncertainty. Finally, the information and data available on the fragility limits of equipments are invariably limited. Moreover, when data are available, there is significant degree of scatter. The analysis of the different sources of uncertainty is discussed below.

Free-Field Ground Shock Environment -- For the reasons stated above, we shall limit the consideration of the free-field shock environment only to the ground accelerations. Available information on free-field ground accelerations has been discussed earlier in Sects. 2.5.2 and 3.4. Accelerations for four types of material, namely, alluvium, dry tuff, wet tuff, and hard rock, were presented in Perret and Bass (1975). Aside from giving the linear regression of the logarithm of ground acceleration on the logarithm of scaled range, values of the associated "variance factors" were also given, on the basis of which the parameter ζ is,

$$\zeta = \ell n \text{ (variance factor)}$$

Thus, the following can be obtained from the results presented in Perret and Bass (1975):

Table 3: Free-Field Acceleration Uncertainty

Material	Scaled Range ($m/KT^{1/3}$)	Variance Factor	ζ_a	c.o.v. δ_a
alluvium	60-350	2.34	0.85	1.03
dry tuff	100-500	2.12	0.75	0.87
wet tuff	30-600	2.21	0.79	0.93
hard rock	90-2200	1.56	0.44	0.46

Table 3 then summarizes the level of uncertainty that can be expected in the prediction of free-field ground shock accelerations; in particular, the last column of the above table shows the c.o.v. ranging from 0.46 for hard rock to 1.03 for alluvium.

Shock Response of Equipment -- The shock environment to which an equipment may be subjected will depend also on where in a structure the equipment is mounted or attached to. For equipments that are mounted onto the floor or an exterior wall of an underground structure, the shock environment may essentially be that of the free-field. However, for those equipments that may be mounted or attached to an intermediate floor or an interior wall, the environment will be the in-structure motions at the location of the equipment, which may be filtered and amplified from that of the free-field motions.

For equipments subject to weapon-induced shock motions, it is reasonable to assume that the uncertainty associated with the response amplification will be comparable to those found in the response prediction of structures subjected to strong-motion earthquakes. In the study of a large number of earthquakes, Mohraz, Hall, and Newmark (1974) reported the 50 and 90-percentile values of the maximum response amplifications for displacement, velocity, and acceleration, giving the following:

	Displacement Amplification	Velocity Amplification	Acceleration Amplification
50% response	1.4	1.66	2.11
90% response	2.21	2.51	2.82

Assuming that the respective percentile amplifications given above were based on a Gaussian distribution, the corresponding c.o.v. are, therefore,

$$\delta_{\alpha_d} = 0.45$$

$$\delta_{\alpha_v} = 0.40$$

$$\delta_{\alpha_a} = 0.26$$

The c.o.v. obtained above would account for the uncertainty associated with the inherent variability of a given ground motion. However, in predicting the response of an equipment, there are other sources of uncertainty or factors that will affect the equipment response; these other sources of uncertainty must also be considered. In short, the final response of an equipment may be given as

$$\text{Response} = N_s N_A N_d \alpha_a a_{FF}$$

where:

N_s = correction for site-dependent factors;

N_A = correction for idealized modeling of system including soil-structure interaction;

and, N_d = correction for damping.

The uncertainty associated with the site-dependent factors, may be assumed to be around 20%; whereas, according to Newmark (1974) the uncertainty associated with damping is of the order of 30%. Finally the uncertainty associated with the modeling of the system may be conservatively assumed to be 30% (Ang and Newmark 1977).

Therefore, for equipments that are essentially subject to free-field environments, the uncertainty in the predicted response would be as follows:

Wet tuff:

$$\Omega_{response} = \sqrt{0.20^2 + 0.30^2 + 0.26^2 + 0.93^2}$$

$$= \sqrt{0.54^2 + 0.93}$$

$$= 1.07$$

$$\zeta_{response} = 0.87$$

Hard rock:

$$\Omega_{response} = \sqrt{0.54^2 + 0.46^2} = 0.71$$

$$\zeta_{response} = 0.64$$

Dry tuff:

$$\Omega_{response} = \sqrt{0.54^2 + 0.87^2}$$

$$= 1.02$$

$$\zeta_{response} = 0.84$$

Alluvium:

$$\Omega_{response} = \sqrt{0.54^2 + 1.03^2}$$

$$= 1.16$$

$$\zeta_{response} = 0.92$$

The results of this uncertainty evaluation may be summarized as follows:

Table 4: Equipment Response to Free-Field Motions

Material	Equipment Response Uncertainty	
	$\Omega_{Response}$	$\zeta_{Response}$
Alluvium	1.16	0.92
Dry tuff	1.02	0.84
Wet tuff	1.07	0.87
Hard rock	0.71	0.64

Equipment Subjected to In-Structure Motions -- For equipments that are attached to or mounted on interior walls or intermediate floors of a structure or building, the shock environment may be significantly different from that of the free-field; the motions are essentially in-structure motions. The analysis or prediction of such in-structure motions at the location of the mounting will contribute further uncertainty. This would include, in particular, the uncertainty associated with the modeling and idealization of the structure for analysis purposes, and the variabilities of the structural mass and stiffness, as well as the error in the usual elastic assumption for materials that behave inelastically in the range of behavior of interest.

Conceivably, the total uncertainty associated with the factors influencing the in-structure motions could be of the order of 20 to 50%. A c.o.v. of 30% may be reasonable. Hence, for equipment subjected to in-structure motions, the response uncertainties should include this c.o.v. of 30%, in addition to those given earlier; the results may then be summarized as follows:

Table 5: Equipment Response to In-Structure Motions

Material	Equipment Response Uncertainty	
	$\Omega_{response}$	$\zeta_{response}$
Alluvium	1.20	0.94
Dry tuff	1.06	0.87
Wet tuff	1.11	0.90
Hard rock	0.77	0.68

Equipment Fragility -- The survivability of an equipment to ground shocks will, of course, depend also on the maximum dynamic response that it can tolerate and remain operational; this limiting response level is known as the fragility limit of the equipment, and may be expressed in terms of the tolerable peak acceleration.

Aside from the maximum motion to which an equipment may be subjected, the fragility is actually dependent also on the nature of the input excitation; in particular, whether it is a transient shock or a vibratory-type motion will influence the fragility limit. Evaluation of the fragility limits of equipment must largely be derived from experimental data. Such data are quite limited, however, and are available only in terms of acceleration tolerance. Ranges of such tolerances have been reported by Newmark, et al (1963); from such data the c.o.v. of the fragility limits of various generic types of equipment may be estimated, assuming (conservatively) uniform PDF's in these ranges. Results of such evaluation can be summarized as shown below in Table 6.

Table 6: Equipment Fragility and Uncertainty

Equipment Classification	Unmounted: Range of Tolerable Acceleration	Unmounted: c.o.v.	Mounted: Range of Tolerable Acceleration	Mounted: c.o.v.
Class A; Heavy Equipment	10-30 g	0.29	20-60 g	0.29
Class B: Medium weight machinery	15-45 g	0.29	30-90 g	0.29
Class C: Light machinery	30-70 g	0.23	50-150 g	0.29
Class D: Communication Equipment	2-8 g	0.35	10-90 g	0.46
Class E: Small electronic equipment	20-80 g	0.35	50-450 g	0.46
Class F: Display tubes	1.5-4.5 g	0.29	5-25 g	0.39
Class G: Transistorized computers	5-20 g	0.35	20-200 g	0.47
Class H: Storage batteries	20-120 g	0.41	50-250 g	0.39

For the purpose of estimating the uncertainties of equipment fragilities, equipments may be divided into machinery and electrical/electronic equipment, and whether or not it is mounted; on this basis, the following average c.o.v. may be used in general.

Table 7: Uncertainty in Equipment Fragility

Equipment Type	c.o.v. $\Omega_{fragility}$: mounted	c.o.v. $\Omega_{fragility}$: unmounted	$\zeta_{fragility}$: mounted	$\zeta_{fragility}$: unmounted
Machinery	0.29	0.27	0.28	0.27
Electrical/electronic equipment	0.43	0.35	0.41	0.34

Equipment Vulnerability to Ground Shocks -- The response spectra specified for the design of equipments may be assumed to be the peak base motions amplified by the respective 90-percentile amplification factors. On this basis, the safety factor underlying the design of equipments would be as follows:

Since the c.o.v. of the acceleration response amplification is $\delta_{\alpha_a} = 0.26$, the design spectral acceleration is

$$A_{design} = (1 + 1.28 \times 0.26)\,\bar{A}$$

$$= 1.33\,\bar{A}$$

where $\bar{A}$ is the mean spectral acceleration.

According to Newmark, et al (1963), the fragility limits (for moderate damage) of *equipments used in design* are approximately the average values within the ranges presented in Table 6; therefore, implicitly, equipments are designed for shock resistance with a mean (or median) safety factor of around 1.33. On this basis and using the average uncertainty information developed in Tables 4, 5 and 7, the survival probabilities of equipments to ground shocks in different materials are as follows:

Table 8: Equipment Survival Probability p_S

Material	Free-Field Motions		In-Structure Motions	
	Machinery	Elec./Elec. Equipment	Machinery	Elec./Elec. Equipment
Alluvium	0.62	0.61	0.62	0.61
Dry tuff	0.63	0.63	0.63	0.62
Wet tuff	0.63	0.62	0.62	0.62
Hard rock	0.67	0.66	0.66	0.65

According to the above results, equipment survivability against moderate damage, therefore, ranges between 61% and 67%.

IV. SUMMARY AND CONCLUSIONS

4.1 Main Emphases of Study

The concepts of probability and statistics that are essential for applications in engineering in general, and strategic structural problems in particular, have been reviewed and presented. It is recognized that although mathematical methods of probabilistic analysis are widely known and available, the implementation of such standard mathematical techniques is limited and not straight-forward. The limitations and difficulties are invariably associated with the lack of sufficient probabilistic information necessary for a rigorous approach. Applications to real or realistic engineering problems must explicitly recognize the existing state of information and take into consideration the role of judgment. In this light, strict mathematical rigor or exactness is not always warranted, especially if such rigor involves major complexity; in view of the fact that subjective judgments are invariably needed to supplement existing state of information, mathematical exactness does not necessarily mean practical validity or credibility of results. Approximations (with resulting simplifications) that are consistent with the quality of available information, therefore, is a more sensible approach to many practical problems.

The necessary approximate methods of probabilistic analysis are developed for the purpose described above. Moreover, the emphasis is on methods that are most useful for the development of probabilistic relationships necessary for probability-based design, in contrast to methods that may be more relevant for probabilistic assessment. In this regard, the concept of expected probability is stressed, as this approach avoids ambiguity that would otherwise arise in methods that are more suited strictly for assessment purposes.

One of the main objectives of an engineering analysis is the development of relationships for design. Design may be the determination of adequate structural resistance to withstand a specified weapon effect, or the determination of an appropriate weapon system to insure the destruction of an enemy facility. In light, or under conditions, of uncertainty the required design can be developed to assure mission success within a given probability of survival or probability of kill. For this purpose, therefore, proper allowance for the effect of uncertainty on design must be provided; i.e. all sources of uncertainty must be included, irrespective of whether it is due to inherent randomness or associated with prediction errors. For this reason, and the resulting unambiguity, the total or expected probability approach is most appropriate for developing probability-based design relationships.

The scope of the present study would not permit the total coverage and exhaustive illustration of all aspects of strategic problems in which probability concepts and methods may be effective. Nevertheless, the methods developed and illustrated herein could serve to demonstrate the feasibility for implementation to other strategic problems. The ease of implementation and clarification of the role of probability were emphasized in the development of the material. Hopefully, this will contribute to the effective use of probability in other areas of strategic problems.

Additional Remarks

Because of the impossibility for absolute assurance of survivability or kill, as a consequence of uncertainty, probability concepts are needed. A statement of probability, therefore, refers to the degree of assurance for survival or kill. The real role of probability and statistics in strategic planning and design, or in engineering in general, therefore, lies in the quantitative framework for uncertainty assessment and analysis of its effects on the performance or effectiveness of an engineering system. This role is supplemental to the existing deterministic approach to engineering. On this premise, the most effective way to implement probability concepts is to develop the required probability-based relationships that can be used within the existing methods of engineering analysis and design.

The tools and concepts presented in the report are limited to analytical methods, or those requiring direct numerical calculation, as opposed to Monte Carlo methods that require repeated sampling calculations. The Monte Carlo method is essentially a process of repeated deterministic calculations, each of which is based on a specific set of values for the pertinent variables in a problem, which are randomly chosen from the respective populations of known or assumed distributions. As a consequence, Monte Carlo calculations can be expensive if used indiscriminately; furthermore, the results of a Monte Carlo calculation applies to a specific problem or condition, and do not permit generalization or extrapolation. Monte Carlo methods, therefore, should be used with some discretion; special situations requiring Monte Carlo calculations would include the following:

(1) When no analytical or nonrepetitive numerical methods are available.

(2) In some cases, the accuracy of approximate analytical techniques cannot be verified except through large-sample Monte Carlo calculations.

For this latter purpose, Monte Carlo simulation is a specially effective tool.

4.2 Some Suggestions for Further Study

The basic probabilistic methodology is now generally available; however, to effectively implement such a methodology in strategic planning and design would require the development of information specifically needed for strategic purposes. In particular, the following studies would be worthy of further efforts:

(1) Establish Credible Information Base for Specific Applications -- In implementing any methodology, the result derived therewith will obviously be only as good as the data and information used in its derivation; in the present case, the required input information must include explicit measures of uncertainty. For this reason, credible measures of uncertainty associated with the major factors and parameters underlying each type or class of strategic problems should be carefully established. In particular, efforts should now be directed to the careful analysis of available data for the purpose of evaluating the uncertainty underlying the inherent randomness of nature, and the systematic documentation of judgments relative to the accuracy of current prediction models from which the associated uncertainties due to model imperfections may be assessed. The importance of

credible uncertainty measures cannot be over-emphasized; the usefulness of a calculated probability of survival (or kill) would clearly depend on the credibility of the uncertainty measures used in its calculation.

Efforts of this type should preferably be devoted to each type or class of strategic problems, in order to sufficiently emphasize the unique characteristics of each problem. Therefore, separate tasks would be necessary to fully develop the required uncertainty information. The objective of such specific studies would be to establish the basis for credible assessment of uncertainty; these bases should be documented systematically and the original sources referenced in such a form that others can review and judge the validity of the outcomes and results.

Each study should carefully and systematically review the current methods for strategic analysis and design and the identification of all the major environmental and system parameters. All known and accessible data pertinent to strategic design should be reviewed and analyzed to form the information base for the respective uncertainty measures. Where data are insufficient or lacking, experts in the fields of strategic planning and design should be consulted for their critical input. The latter may be obtained through one or more meetings of groups of experts to derive consensus subjective assessments in those areas where purely judgmental opinions are required.

(2) Determination of Systems Probability -- This study should be a concerted effort to develop approximate methods for calculating the probability of survival or failure of a system as a function of its components. The problem is one of evaluating the probability of a system on the basis of the corresponding probabilities of its separate components, taking into account the significance of the system configuration (e.g. series or parallel systems, or combinations thereof, as well as systems that may not be classified into simple series and parallel components), and the effect of correlations between the components.

The study should include an identification of the major types of strategic structures and facilities, and develop appropriate calculational tools without having to resort to brute-force Monte Carlo calculations or simulation unless necessary.

(3) Application of Statistical Decision Theory -- In a broader sense, a calculated probability of survival (or kill) is intended to provide information on the significane of uncertainty for the purpose of decision making. From this standpoint, the theory and tools of statistical decision are relevant. This study should include a review of the basic concepts of the current statistical decision theory with a view toward its potential role and applications in the planning and design of strategic systems. The study may include targeting and the planning of experiments, and should identify the most useful elements of the theory of statistical decision and develop additional tools as necessary. Specific utility functions may be reviewed and their relevancy to strategic problems should be identified. The specific tasks of this study would include the following:

(a) Review and describe the state-of-the-art of statistical decision theory, including the relevancy of specific utility functions.

(b) Identify the decision criteria in strategic defense applications.

(c) Develop additional tools as necessary to implement, or facilitate the implementation of, statistical decision analysis to strategic systems.

(d) Illustrate the implementation of statistical decision analysis to specific problems including, for example, the planning of targeting decision, defense survivability design, and in test planning including sequential (or staged) planning of experiments.

(4) Development of Probabilistic Software for Strategic Purposes -- This study will require the development of general-purpose computer codes for strategic purposes. It should include the calculation of the probability of survival or kill, for general types of probability distributions. Except for special types of probability distributions, such as the lognormal, numerical integration would generally be necessary in the evaluation of the probability of survival or kill.

Computer codes to perform numerical integrations for derived probability distributions, such as the probability distribution of a function of several random variables, would also be required. Finally, computer codes for evaluating the survival probability of a general strategic system, on the basis of probabilistic information of its components, are also needed.

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Appendix: DEFINITION OF SYMBOLS

The following is a list of the key symbols used in the report.

R = Resistance or strength

S = Load or load effect

$f_R(r)$, $f_S(s)$ = Probability density functions (PDF) of R and S, respectively.

$F_R(r)$, $F_S(s)$ = Cumulative distribution functions (CDF) of R and S.

p_F = Probability of failure

p_S = Probability of survival

$\tilde{r}$, $\tilde{s}$ = Median values of R and S, respectively

$\bar{r}$, $\bar{s}$ = Estimated mean values of R and S, respectively

μ_R, μ_S = Actual or true mean of R and S.

σ_R, σ_S = Standard deviations of R and S.

ζ_R^2, ζ_S^2 = Variance of ℓn R and ℓn S, respectively.

$\tilde{\theta}$ = Median safety factor, ratio $\tilde{r}/\tilde{s}$

$\bar{\theta}$ = Mean safety factor, ratio μ_R/μ_S.

$\Phi(x)$ = CDF of the standard normal distribution

$\beta = \Phi^{-1}(p_S)$, the "safety index".

ν = mean bias or systematic error of modeling and prediction.

δ = c.o.v. representing measure of uncertainty due to inherent randomness

Δ = c.o.v. representing measure of uncertainty associated with dispersive error of modeling and prediction.

$\Omega = \sqrt{\delta^2 + \Delta^2}$, total c.o.v.

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